Fundamentals in Biophotonics

Photon /wave particle duality

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Blackbody radiation

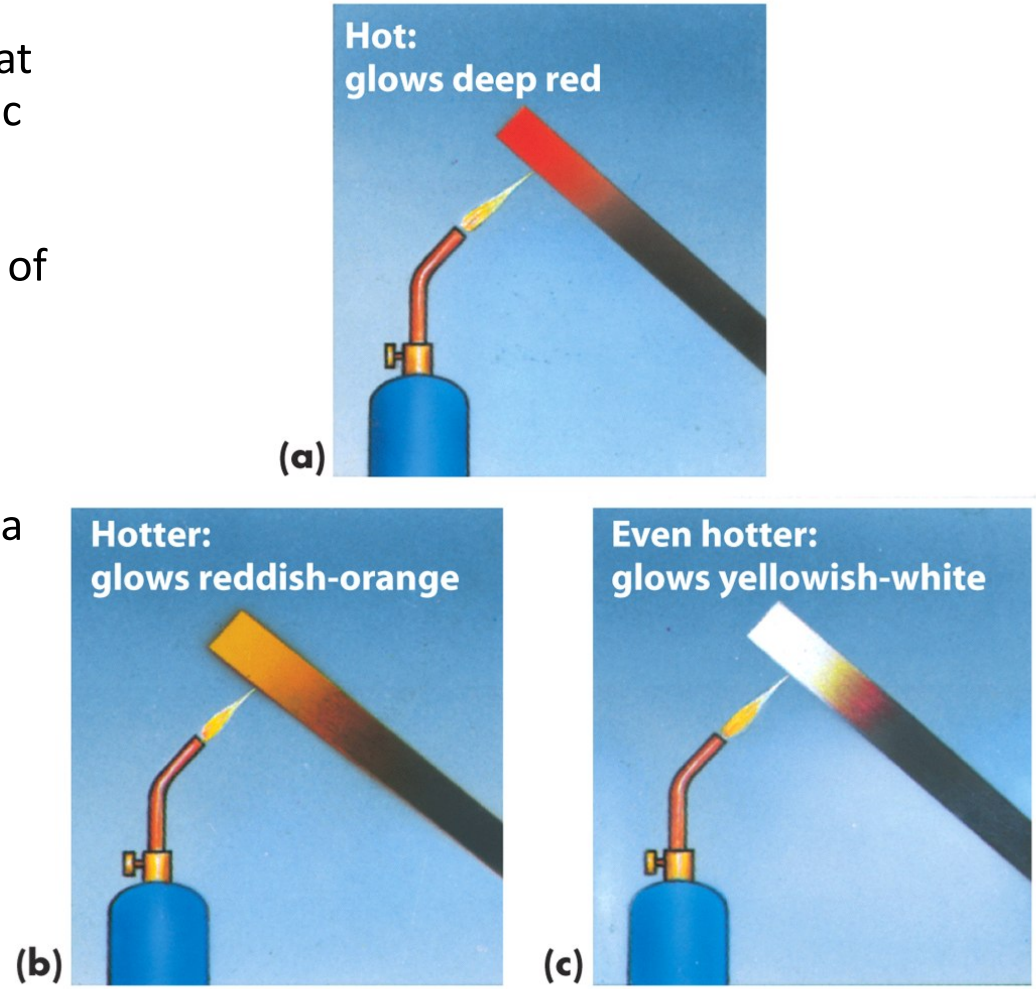
- Hot objects glow (toaster coils, light bulbs, the sun).

- As the temperature increases the color shifts from Red (700 nm) to Blue (400 nm)

A blackbody is a hypothetical object that is a perfect absorber of electromagnetic radiation at all wavelengths

Stars closely approximate the behavior of blackbodies, as do other hot, dense objects

The intensities of radiation emitted at various wavelengths by a blackbody at a given temperature are shown by a blackbody curve



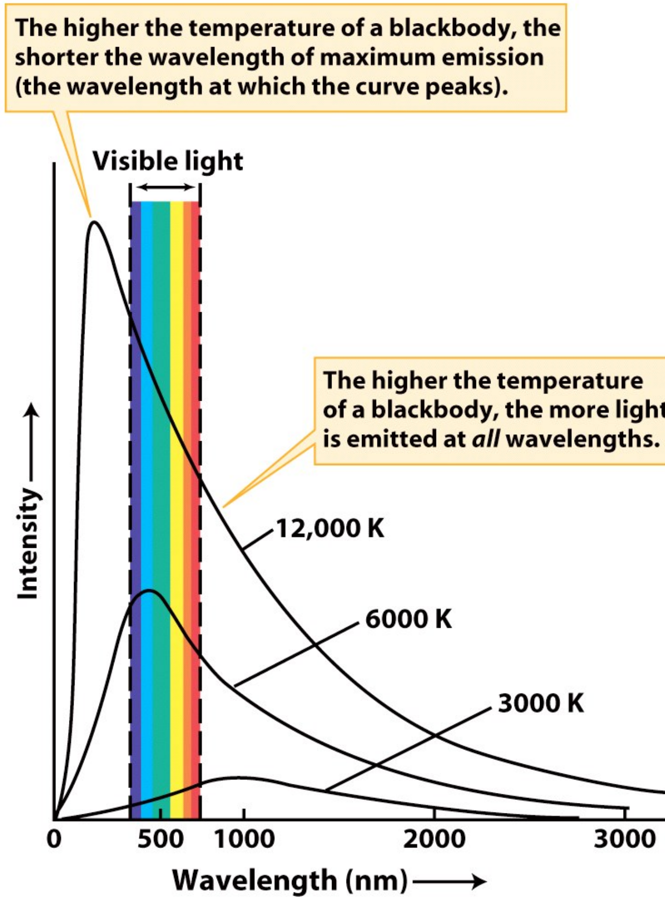
Wien's Law

$$\lambda_{max} = \frac{0.0029 \text{ K m}}{T}$$

λ_{max} = wavelength of maximum emission of the object (in meters)

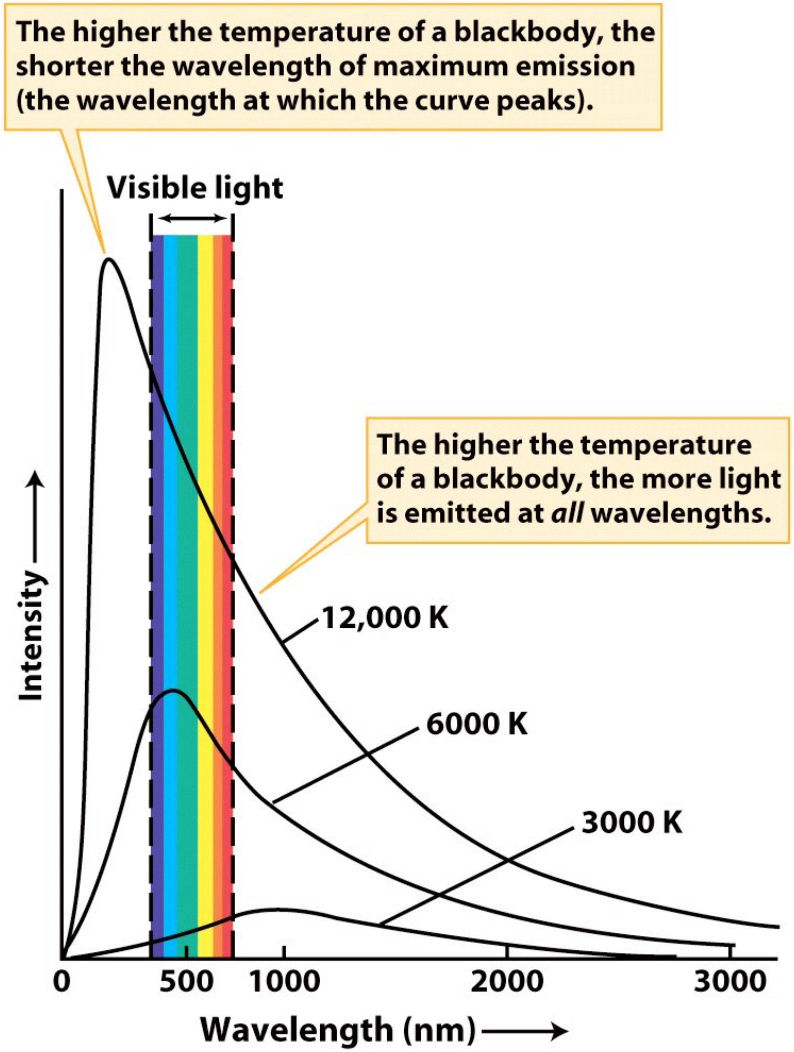
T = temperature of the object (in kelvins)

Wien's law states that the dominant wavelength at which a blackbody emits electromagnetic radiation is inversely proportional to the Kelvin temperature of the object



Black body radiation

- The classical physics prediction was completely wrong! (It said that an infinite amount of energy should be radiated by an object at finite temperature)



Wien's law

Rayleigh–Jeans law

$$I(\lambda, T) = \frac{2\pi c k T}{\lambda^4}$$

Stefan-Boltzman law

$$j = \sigma T^4$$

Max Planck found he could explain these curves if he assumed that electromagnetic energy was radiated in discrete chunks, rather than continuously.

The "quanta" of electromagnetic energy is called the photon.

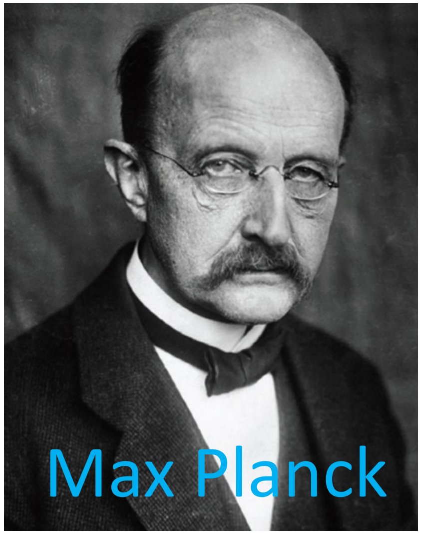
Energy carried by a single photon is

$$E = h\nu = \hbar\omega,$$

Planck's constant: h = 6.626 x 10^{-34} Joule sec

Planck's Theory of Blackbody Radiation

- 1858 – 1847
- German physicist
- Introduced the concept of "quantum of action"
- In 1918 he was awarded the Nobel Prize for the discovery of the quantized nature of energy.



- In 1900 Planck developed a theory of blackbody radiation that leads to an equation for the intensity of the radiation.

- This equation is in complete agreement with experimental observations.

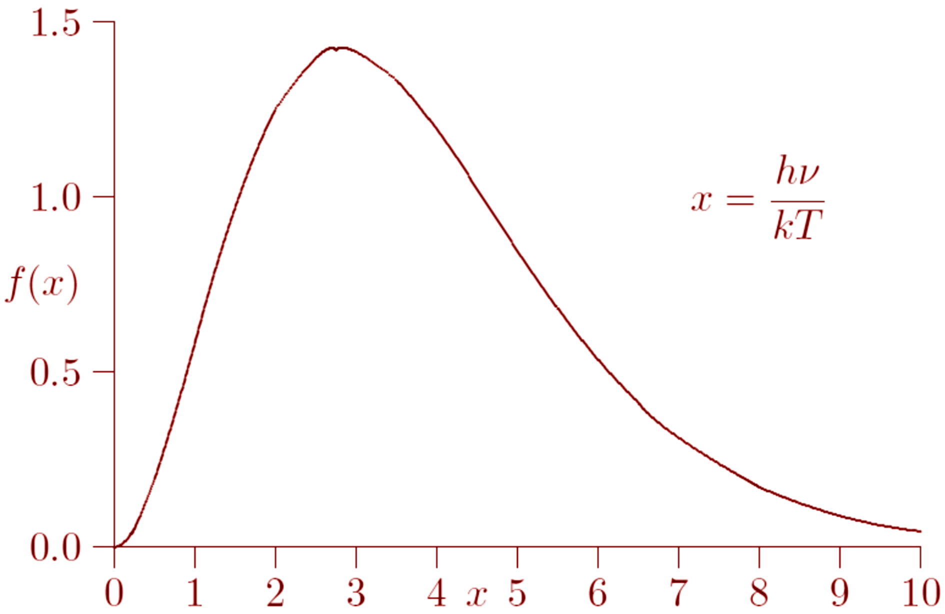
- He assumed the cavity radiation came from atomic oscillations in the cavity walls.

- Planck made two assumptions about the nature of the oscillators in the cavity walls.

Planck's Theory of Blackbody Radiation

- we demonstrate why quantum concepts are necessary to account for **Planck Blackbody radiation formula**. The programme to derive this formula is as follows.

$$f(x)\,\mathrm{d}x \propto \frac{x^3}{\mathrm{e}^x - 1}\,\mathrm{d}x$$



$$u(\nu)\,\mathrm{d}\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{(\mathrm{e}^{h\nu/kT} - 1)}\,\mathrm{d}\nu$$

First, we consider the properties of waves in a box and work out an expression for the radiation spectrum in thermal equilibrium at temperature T .

Planck’s Theory of Blackbody Radiation

- First, we consider the properties of waves in a box and work out an expression for the radiation spectrum in thermal equilibrium at temperature T.

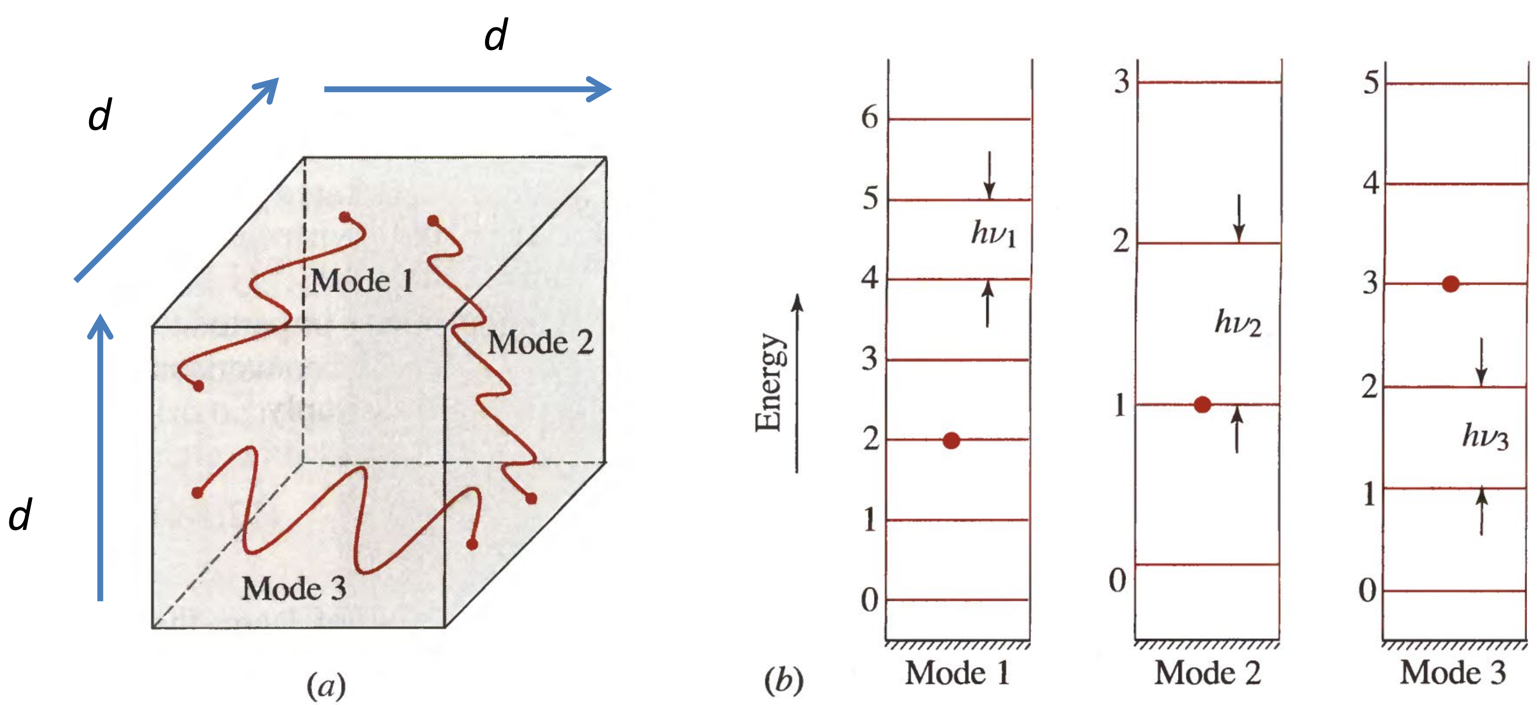
- Application of the law of equipartition of energy leads to the *ultraviolet catastrophe*, which shows that something is seriously wrong with the classical argument.
- Then, we introduce Einstein's deduction that light has to be quantised in order to account for the observed features of the *photoelectric effect*.

- Finally, we work out the mean energy per mode of the radiation in the box assuming the radiation is quantised. This leads to Planck's radiation formula.

- This calculation indicates clearly the necessity of introducing the concepts of quantisation and quanta into physics.

Electromagnetic-Optics Theory of Light in a Resonator

- The concept of the photon and the rules of photon optics are introduced by considering light inside an optical resonator (cavity). This is a convenient choice because it restricts the space under consideration to a simple geometry. However, the presence of the resonator turns out not to be an important restriction in the argument; the results can be shown to be independent of its presence.



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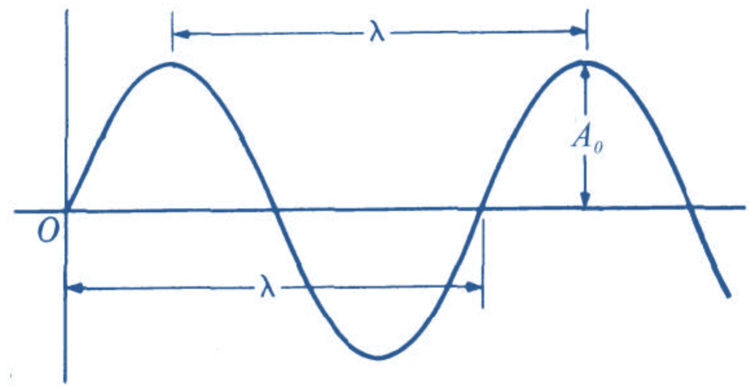
Waves in a Box

- Let us revise the expression for an electromagnetic (or light) wave travelling at the speed of light in some arbitrary direction say, in the direction of the vector *r*.
- If the wave has wavelength λ, at some instant the amplitude of the wave in the *r*-direction is



Wave vector

$$A(r) = A_0 \sin \frac{2\pi r}{\lambda}$$

$$|\boldsymbol{k}| = 2\pi/\lambda.$$

$$A(r) = A_0 \sin(\boldsymbol{k} \cdot \boldsymbol{r}) = A_0 \sin kr,$$

The wave travels at the speed of light *c* in the *r*-direction and so, after time *t*, the whole wave pattern is shifted a distance *ct* in the positive *r*-direction and the pattern is *A0* sin *kr0*, where we have shifted the origin to the point *ct* along the *r*-axis such that *r* = *r0* + *ct*. **Thus, the expression for the wave after time *t* is**

$$A(r, t) = A_0 \sin kr' = A_0 \sin(kr - kct).$$

Waves in a Box

- But, if we observe the wave at a fixed value of ***r***, we observe the amplitude to oscillate at frequency *v*. Therefore, the time dependence of the wave amplitude is $\sin(2\pi t/T)$ where $T = \nu^{-1}$ is the period of oscillation of the wave.

- Therefore, the time dependence of the wave at any point is $\sin \omega t,$ where $\omega = 2\pi\nu$ is the ***angular frequency*** of the wave. Therefore, the expression for the wave

$$A(r,t) = A_0 \sin(kr - \omega t),$$

- and the speed of the wave is $c = \omega/k.$

Consider a cubical box of side ***L*** and imagine waves bouncing back and forth inside it. The box has fixed, rigid, perfectly conducting walls. Therefore, the electric field of the electromagnetic wave *must be zero at the walls of the box* and so we can ***only fit waves into the box which are multiples of half a wavelength.***

Electromagnetic Modes in a Box

- In the ***x*-direction**, the wavelengths of the waves which can be fitted into the box are those for which

$$\frac{l\lambda_x}{2} = L$$

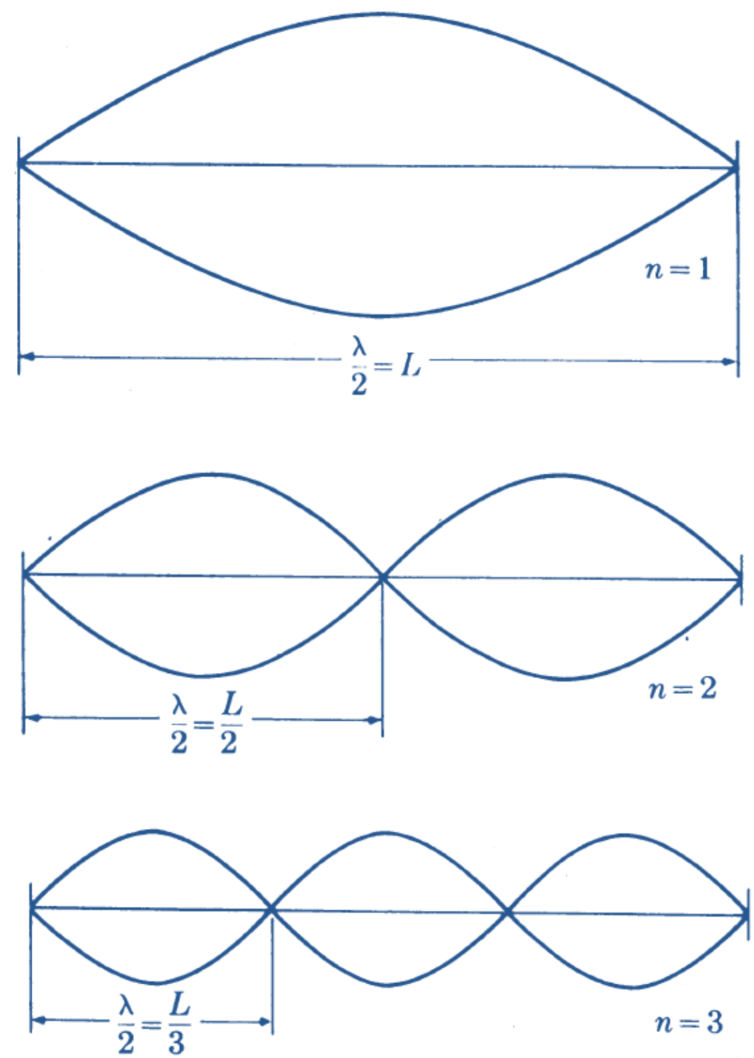
- where *l* takes any positive integral value, 1, 2, 3, ……...Similarly, for the ***y*** and ***z*** directions,

$$\frac{m\lambda_y}{2} = L \quad \text{and} \quad \frac{n\lambda_z}{2} = L,$$

- where *m* and *n* are also positive integers. The expression for the waves which ¯t in the box in the ***x***-direction is

$$A(x) = A_0 \sin k_x x$$

Waves which can be fitted into a box with perfectly conducting walls.



Electromagnetic Modes in a Box

- Now $k_x = 2\pi/\lambda_x$ x is the component of the wave-vector of the mode of oscillation in the *x*-direction. Hence, the values of *kx* which fit into the box are those for which $\lambda_x = 2L/l$ ¸and so
$$k_x = \frac{2\pi l}{2L} = \frac{\pi l}{L},$$
- where *l* takes the values *l* = 1*;* 2*;* 3 *: : :* .
- Similar results are found in the *y* and *z* directions:
$$k_y = \frac{\pi m}{L}, \qquad k_z = \frac{\pi n}{L}.$$
- Let us now plot a three-dimensional diagram with axes *kx*, *ky* and *kz* showing the allowed values of *kx*, *ky* and *kz*. These form a regular cubical array of points, each of them defined by the three integers, *l; m; n*
- This is exactly the same as the velocity, or momentum, space which we introduced for particles.

Electromagnetic Modes in a Box

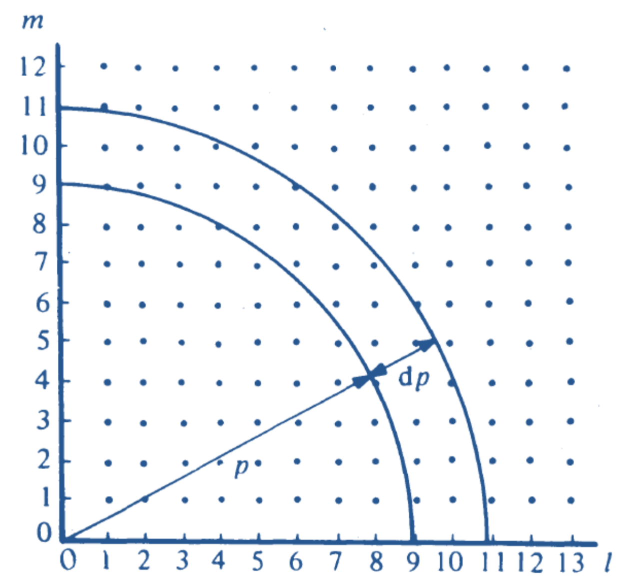
- The waves can oscillate in three dimensions but the components of their *k*-vectors, *kx*, *ky* and *kz*, must be such that they are associated with one of the points of the
- lattice in *k*-space. A wave oscillating in three dimensions with any of the allowed values of *l; m; n* ***satisfies the boundary conditions and so every point in the lattice represents a possible mode of oscillation of the waves within the box, consistent with the boundary conditions***.

Thus, in three-dimensions, the modes of oscillation can be written

$$A(x, y, z) = A_0 \sin(k_x x) \sin(k_y y) \sin(k_z z)$$

To find the relation between ***kx; ky; kz*** and the angular frequency ω of the mode, we insert this trial solution to the three dimensional wave equation

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2}.$$



Illustrating the values **of *l* and *m* which result in components of wave vectors** which can fit in the box.

Electromagnetic Modes in a Box

- The time dependence of the wave is also sinusoidal, and so we can find the *dispersion relation* for the waves, that is, the relation between *!* and *kx; ky; kz*, by substituting the trial solution into

$$|\boldsymbol{k}|^2 = (k_x^2 + k_y^2 + k_z^2) = \frac{\omega^2}{c^2}.$$

- where ***k*** is the three-dimensional wave-vector. Now,

$$k^2 = k_x^2 + k_y^2 + k_z^2 = \frac{\pi^2}{L^2}(l^2 + m^2 + n^2),$$

- and so

$$\frac{\omega^2}{c^2} = \frac{\pi^2}{L^2}(l^2 + m^2 + n^2) = \frac{\pi^2 p^2}{L^2}$$

- where

$$p^2 = l^2 + m^2 + n^2.$$

- We need the count number of modes of oscillation in the frequency interval ν to $\nu + \mathrm{d}\nu$. This is now straightforward since we need only count up the number of lattice points in the interval of *k*-space *k* to *k* + d*k* corresponding to ν to $\nu + \mathrm{d}\nu$.

Electromagnetic Modes in a Box

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Electromagnetic Modes in a Box

- The number density of lattice points is one per unit volume of (*l; m; n*) space. We are only interested in positive values of *l*, *m* and *n* and so we need only consider one-eighth of the sphere of radius *p*. The volume of a spherical shell of radius *p* and thickness d*p* is $4\pi p^2 \, \mathrm{d}p$ and so the number of modes in the octant is

$$\mathrm{d}N(p) = N(p)\,\mathrm{d}p = \left(\frac{1}{8}\right) 4\pi p^2 \,\mathrm{d}p.$$

- Since $k = \pi p / L$ and $\mathrm{d}k = \pi \,\mathrm{d}p / L,$ we find

$$\mathrm{d}N(p) = \frac{L^3}{2\pi^2} k^2 \,\mathrm{d}k.$$

- But $L^3 = V$ is the volume of the box $k = 2\pi\nu/c.$ Therefore, we can rewrite the expression

$$\mathrm{d}N(p) = \frac{V}{2\pi^2} k^2 \,\mathrm{d}k = \frac{V}{2\pi^2} \frac{8\pi^3\nu^2}{c^3} \,\mathrm{d}\nu = \frac{4\pi\nu^2 V}{c^3} \,\mathrm{d}\nu$$

The Average Energy per Mode and the Ultraviolet Catastrophe

- Finally, for electromagnetic waves, we are always allowed two independent modes, or polarisations, per state and so we have to multiply the result by two. Because of the nature of light waves, there are two independent states associated with each lattice point (l, m, n). The final result is that the number of modes of oscillation in the frequency interval ν to $\nu + \mathrm{d}\nu$ is

$$\mathrm{d}N = \frac{8\pi\nu^2 V}{c^3}\,\mathrm{d}\nu$$

- Thus, per unit volume, the number of states is

$$\mathrm{d}N = \frac{8\pi\nu^2}{c^3}\,\mathrm{d}\nu$$

We now introduce the idea that the waves are in thermodynamic equilibrium at some temperature *T*. We showed that, in thermal equilibrium, we award $\frac{1}{2}kT$ of energy to each degree of freedom. This is because, if we wait long enough, there are processes which enable energy to be exchanged between the apparently independent modes of oscillation.

The Average Energy per Mode and the Ultraviolet Catastrophe

- Thus, if we wait long enough, each mode of oscillation will attain the same average energy *E*, when the system is in thermodynamic equilibrium. Therefore, the energy density of radiation per unit frequency interval per unit volume is

$$\mathrm{d}u = u(\nu)\,\mathrm{d}\nu = \frac{8\pi\nu^2}{c^3}\,\overline{E}\,\mathrm{d}\nu,$$

$$u(\nu) = \frac{8\pi\nu^2}{c^3}\,\overline{E}.$$

The Ultraviolet Catastrophe

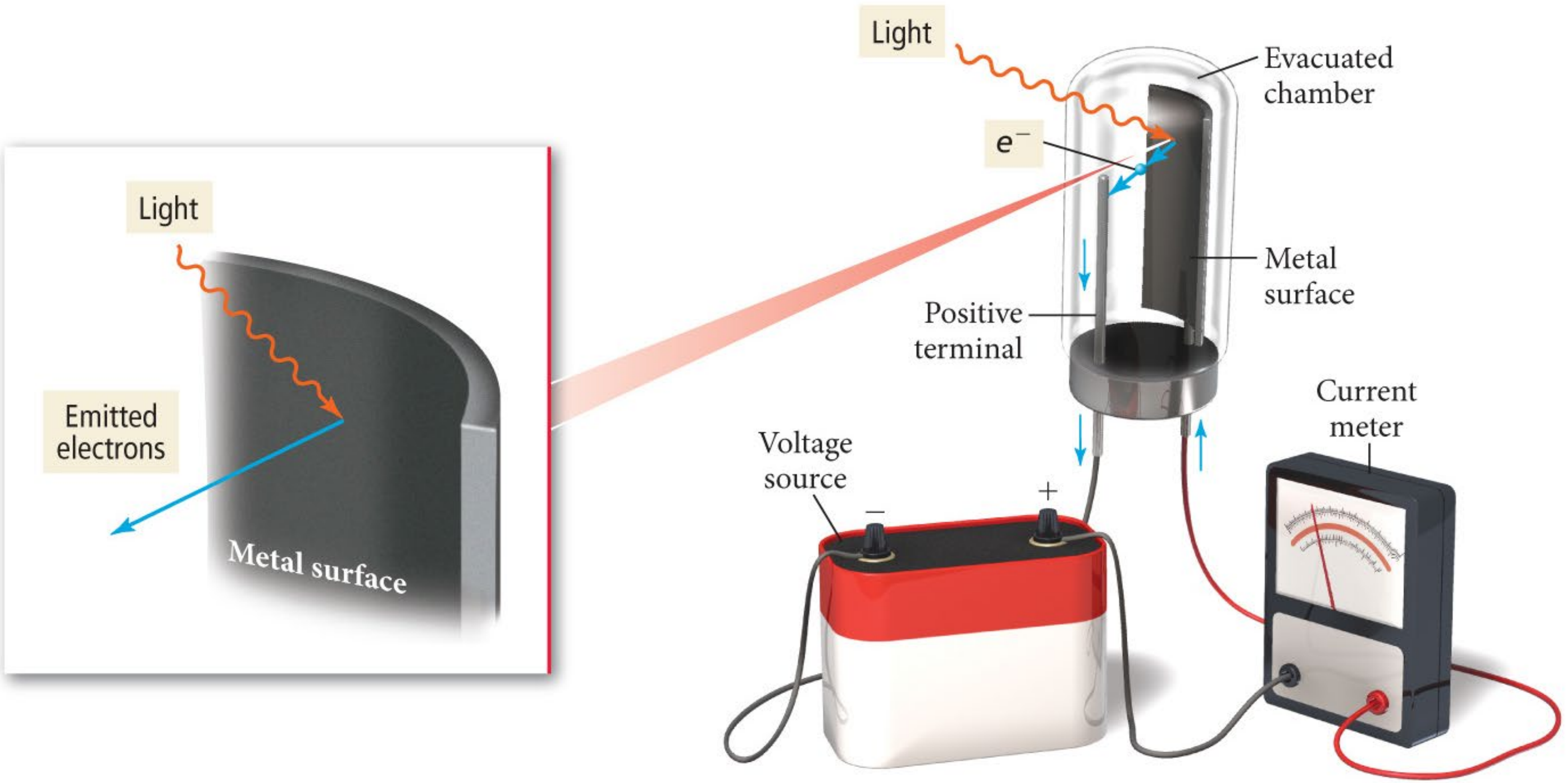
The average energy of a harmonic oscillator in thermal equilib- rium is *E* = *kT* and so the spectrum of black-body radiation is expected to be

$$u(\nu) = \frac{8\pi\nu^2}{c^3}\,\overline{E} = \frac{8\pi\nu^2 kT}{c^3}. \qquad \int_0^\infty u(\nu)\,\mathrm{d}\nu = \int_0^\infty \frac{8\pi\nu^2 kT}{c^3}\,\mathrm{d}\nu \to \infty.$$

The Rayleigh-Jeans Law

The Ultraviolet Catastrophe

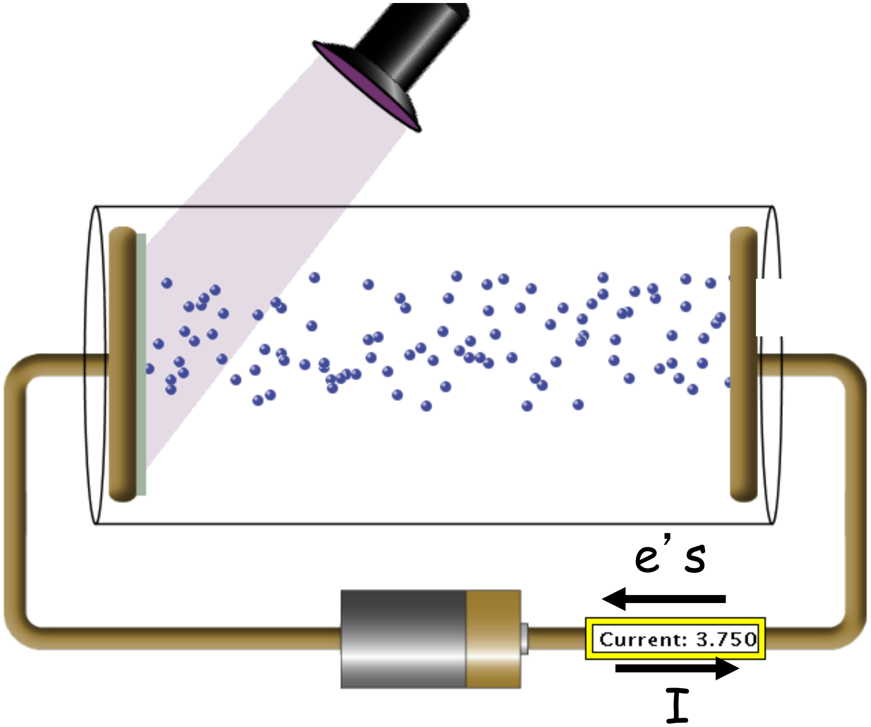
Photoelectric effect

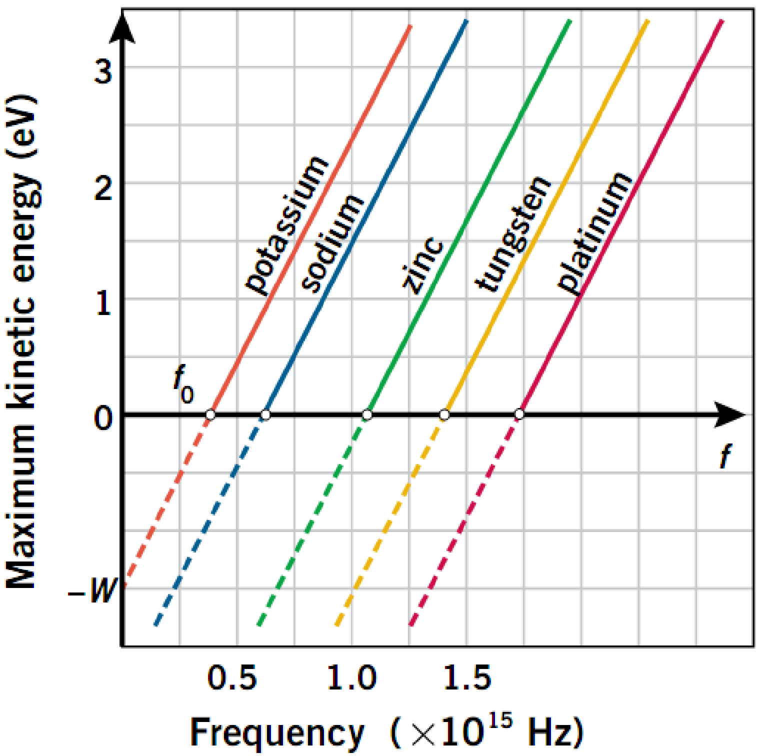


If optical radiation is incident upon a surface, electrons are ejected, provided the ***frequency of the radiation is high enough***. When *monochromatic light*, that is, light of a single frequency, is shone upon the cathode of a discharge tube, it was found that a current flowed between the cathode and the anode, associated with the drift of the ejected electrons to the anode.

Photoelectric effect

There is a minimum frequency below which the light cannot kick out electrons… even if wait a long time





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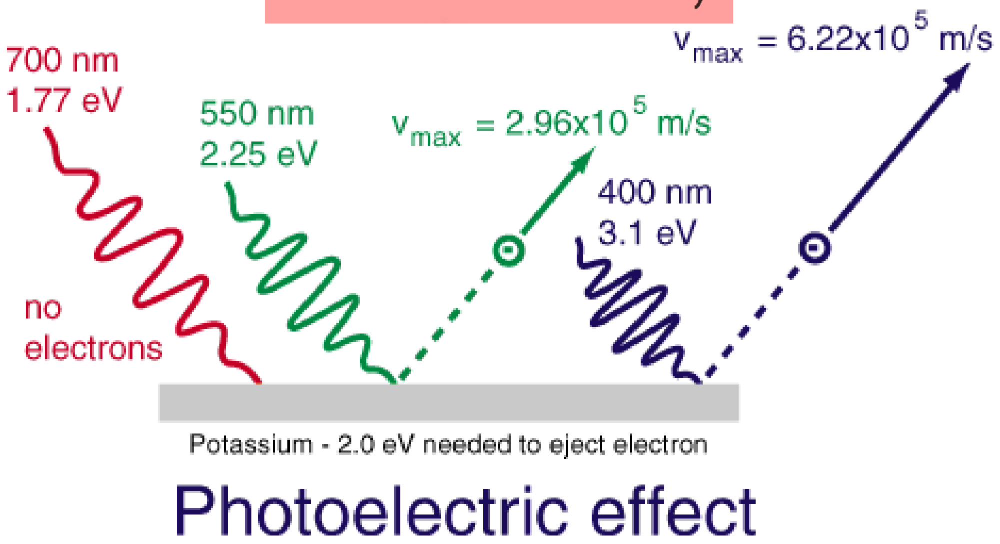
Threshold Frequency (Energy)

- Photon optics provides that the energy of an electromagnetic mode is quantized to discrete levels separated by the energy of a photon . The energy of a photon in a mode of frequency v is

$$E = h\nu = \hbar\omega,$$

A mode containing zero photons carries some energy which is called the **zero-point energy.**



TOTAL energy of n photons

$$E_n = (n + \tfrac{1}{2})\, h\nu, \qquad n = 0, 1, 2, \ldots.$$

- This was the first example of the fundamental phenomenon in physics known as the *wave-particle duality*. It is the statement that, in physics, waves have particle properties and. This lies at the very heart of quantum mechanics and leads to all sorts of nonntuitive phenomena.

Photon energy for infrared photon

- The order of magnitude of photon energy- EXAMPLE Infrared photon

$$E = h\nu = \hbar\omega,$$

$$\lambda = \ 1 \ \mu\text{m} \rightarrow \nu = \frac{c}{\lambda} = 3\times10^{14}\,Hz$$

$$E = h\nu = 1.99\times10^{-19}\,J = (1.99\times10^{-19} / 1.6\times10^{-19})eV$$

$$E = 1.24eV$$

$$E\ (\text{eV}) = \frac{1.24}{\lambda_o\ (\mu\text{m})}.$$

The photoelectric effect demonstrates that light waves have particle properties and that the light quanta, or *photons*, of a particular frequency ν each have energy $h\nu$. We need to reconcile this picture with the classical picture of electromagnetic waves in a box. In the classical picture, the energy associated with the waves is stored in the oscillating electric and magnetic fields.

We found it necessary to impose the constraint that only certain modes are permitted by the boundary conditions { the waves are constrained to ¯t into the box with whole numbers of half wavelengths in the *x; y; z*

Derivation of Planck's law

- Energy of a particular mode of frequency cannot have any arbitrary value but only those energies which are multiples of *hv*, in other words the energy of the mode is $E_n = nh\nu$, where we associate *n* photons with this mode.
- To establish equilibrium, *there must be ways of ex-changing energy between the modes (and photons)* nd this can occur through interactions with any particles or oscillators within the volume or with the walls of the enclosure.

- We now use the **Boltzmann distribution** to determine the expected occupancy of the modes in thermal equilibrium. The probability that a single mode has energy

$$E_n = nh\nu$$

- Boltzmann factor

$$p(n) = \frac{\exp\left(-E_n/kT\right)}{\sum_{n=0}^{\infty} \exp\left(-E_n/kT\right)},$$

Derivation of Planck's law

- The mean energy of the mode of frequency ν is therefore:

$$\overline{E} = \frac{h\nu}{e^{h\nu/kT} - 1}$$

$$\overline{E_\nu} = \sum_{n=0}^{\infty} E_n \, p(n) = \frac{\sum_{n=0}^{\infty} E_n \, \exp\left(-E_n/kT\right)}{\sum_{n=0}^{\infty} \exp\left(-E_n/kT\right)}$$

$$= \frac{\sum_{n=0}^{\infty} nh\nu \exp\left(-nh\nu/kT\right)}{\sum_{n=0}^{\infty} \exp\left(-nh\nu/kT\right)}$$

Thus, for small values of $h\nu/kT$,

$$e^{h\nu/kT} - 1 = \frac{h\nu}{kT}$$

$$\overline{E} = \frac{h\nu}{e^{h\nu/kT} - 1} = \frac{h\nu}{\epsilon/kT} = kT$$

Thus, if we take the ***classical limit***, we recover ***exactly the expression for the average energy of a harmonic oscillator in thermal equilibrium***,

$$\overline{E} = kT.$$

- We can now complete the determination of Planck's radiation formula. We have already shown that the number of modes in the frequency interval to $\nu + d\nu$ is $(8\pi\nu^2/c^3)\, d\nu$ per unit volume. The energy density of radiation in this frequency range is .

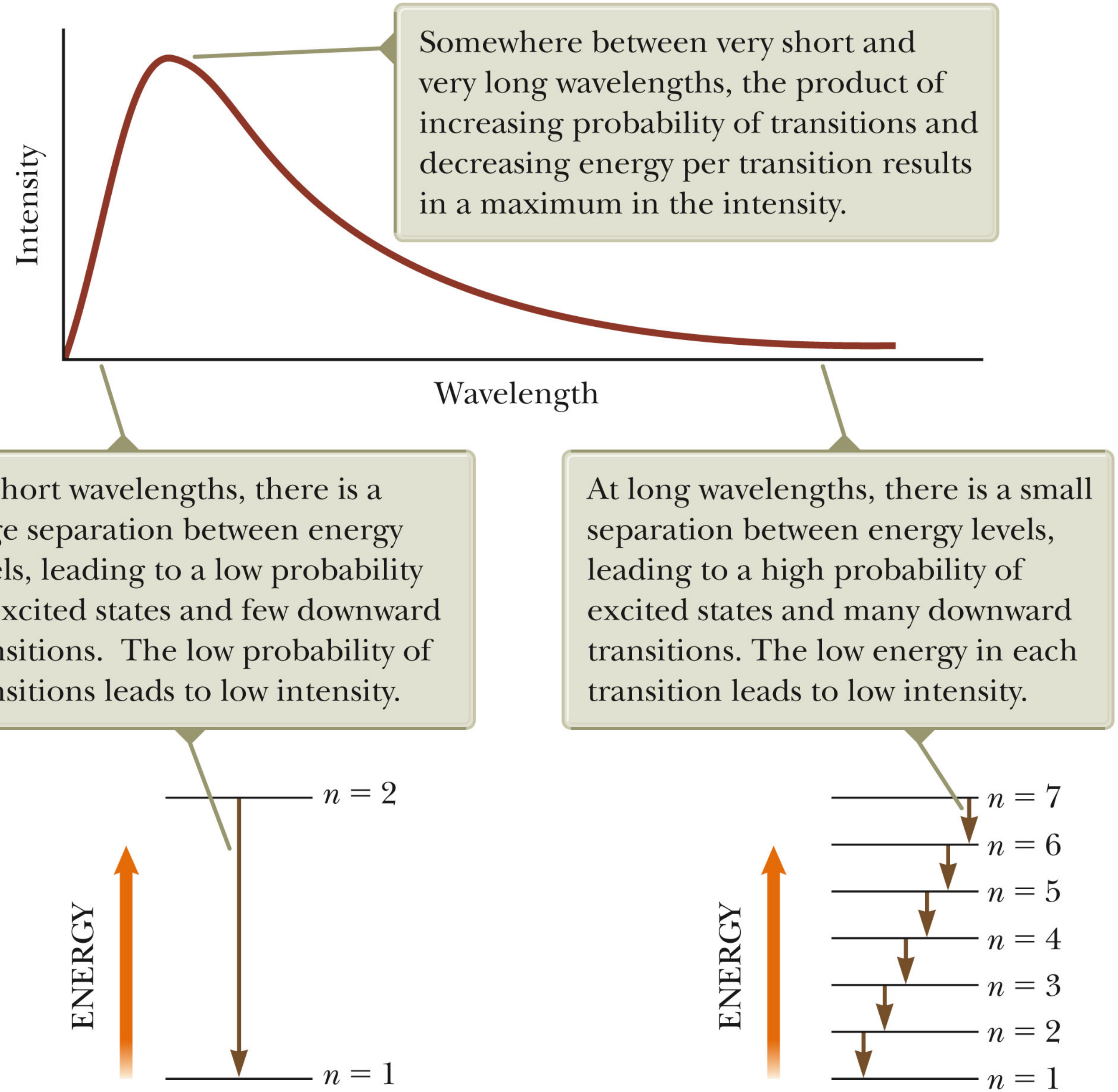
Planck distribution function

$$
\begin{aligned}
u(\nu)\, d\nu &= \frac{8\pi\nu^2}{c^3}\overline{E_\nu}\, d\nu \\
&= \frac{8\pi h\nu^3}{c^3}\frac{1}{\exp(h\nu/kT) - 1}\, d\nu .
\end{aligned}
$$

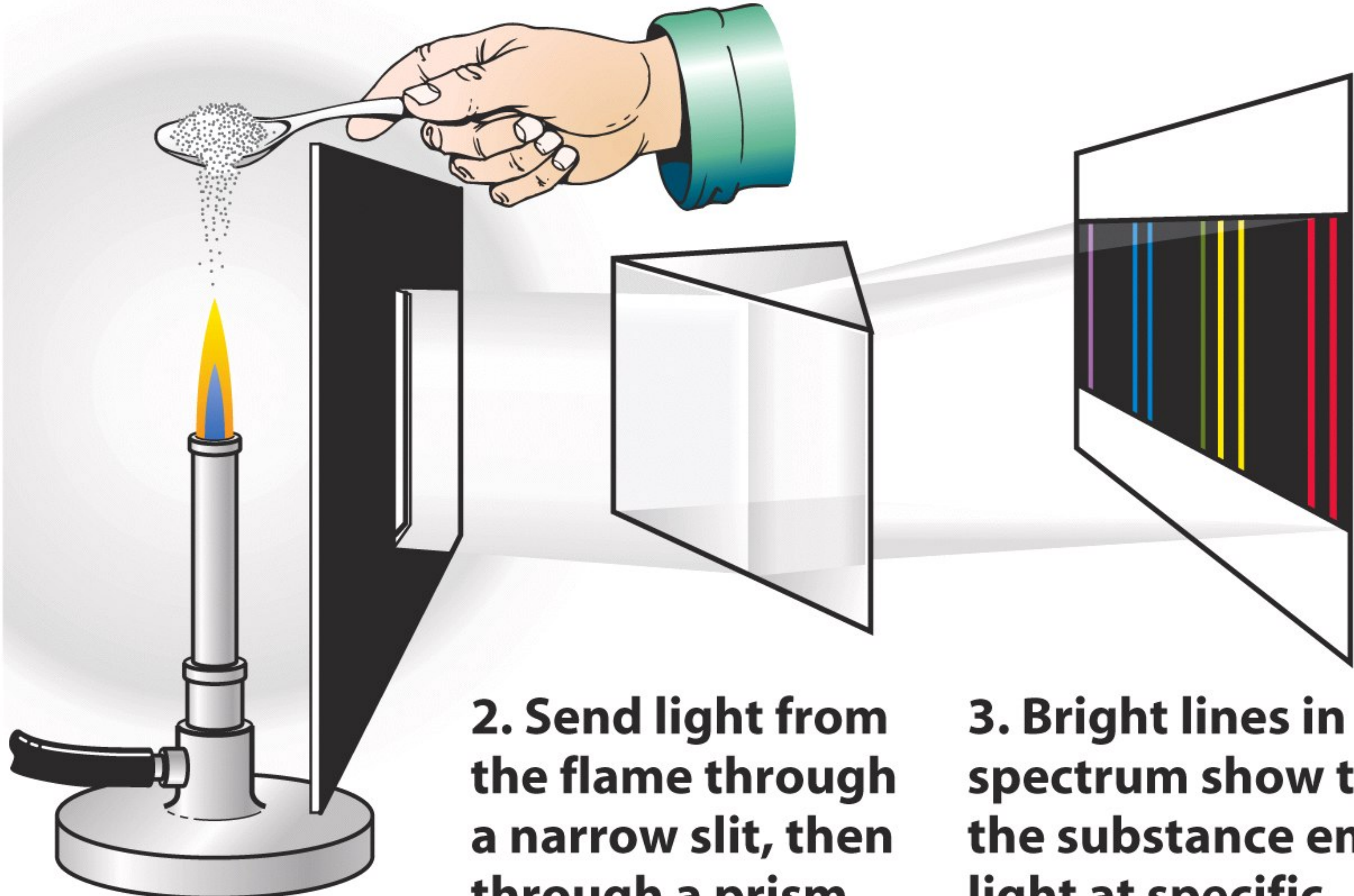
We have made enormous progress. We have introduced ***quantisation, quanta*** and the first example of the *wave-particle duality*. The key idea is that we cannot understand the form of the black-body spectrum without introducing the idea that light consists of energy packets, photons, with energies

$$E = h\nu = \hbar\omega,$$

Planck's Model, Graph



Chemist Observation-What is a photon a how light is generated ?



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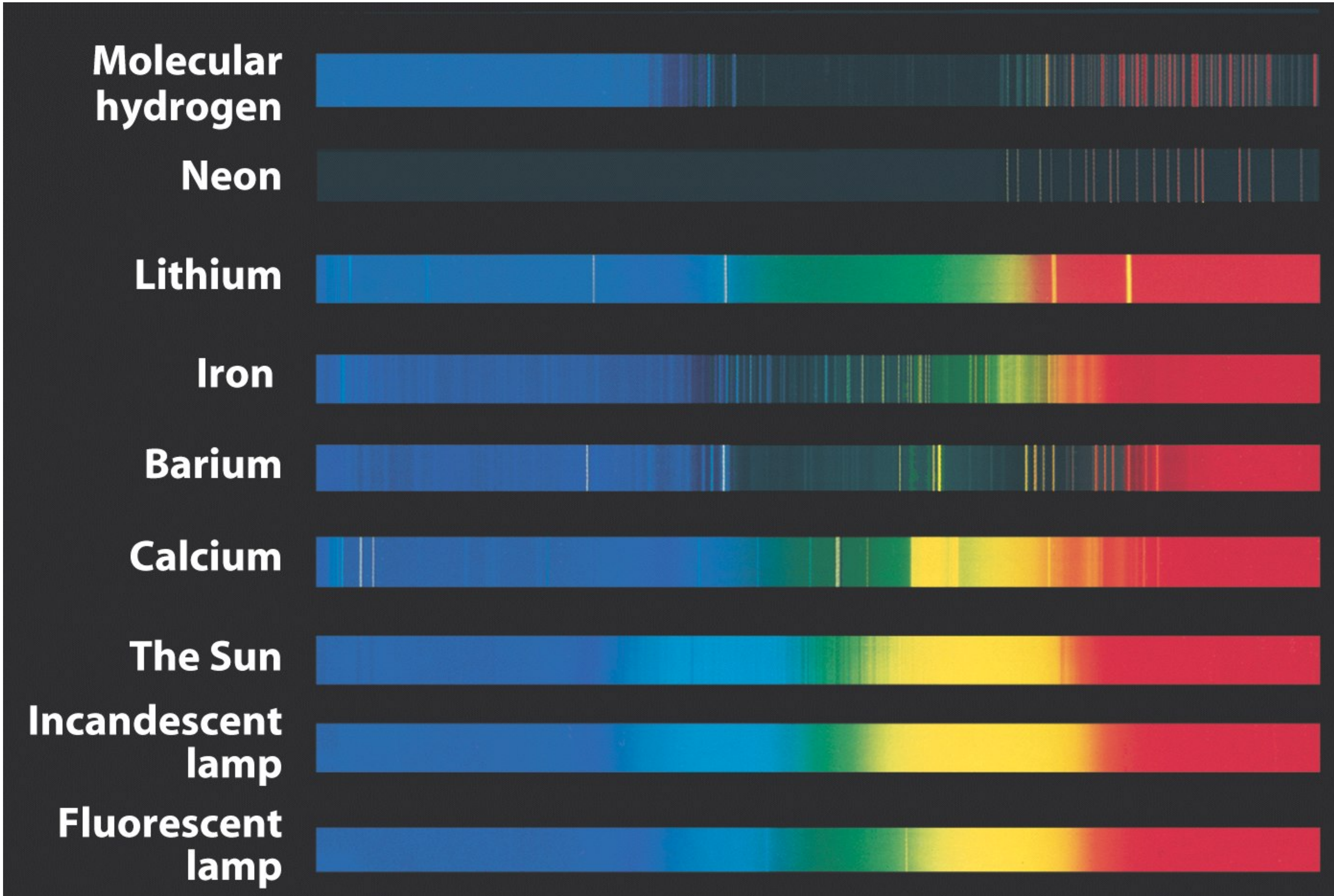
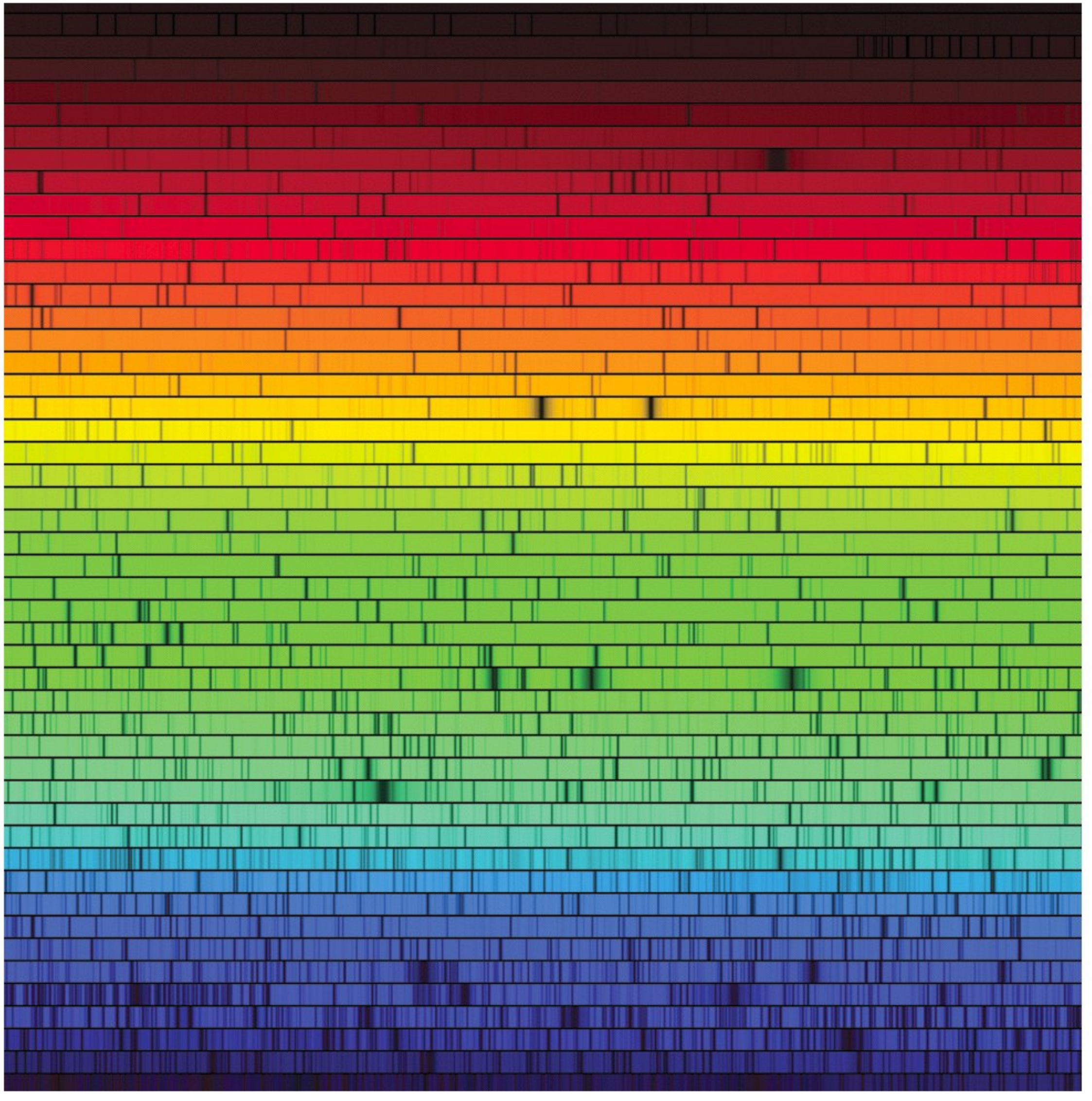
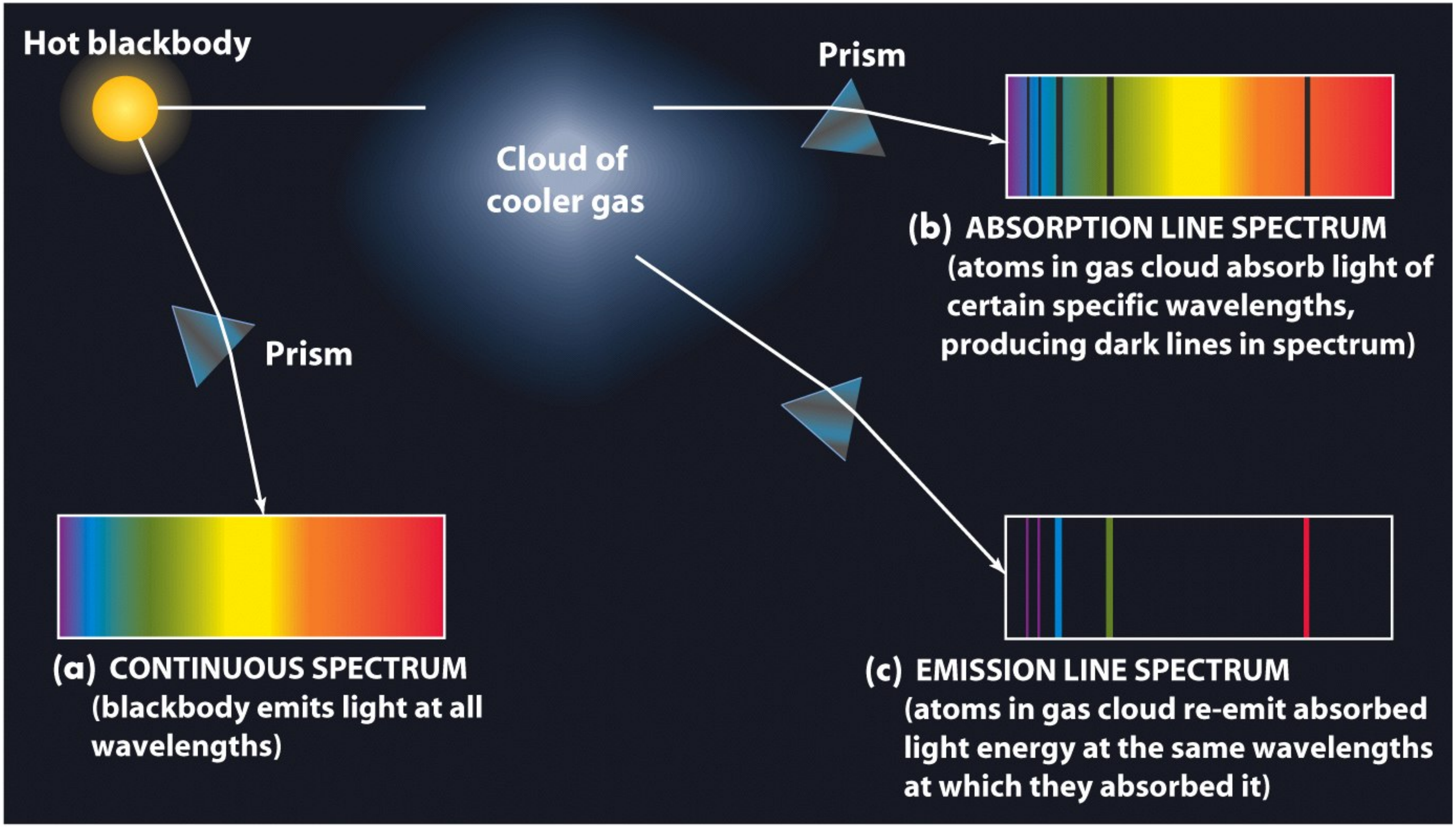


Each chemical element produces its own unique set of spectral lines

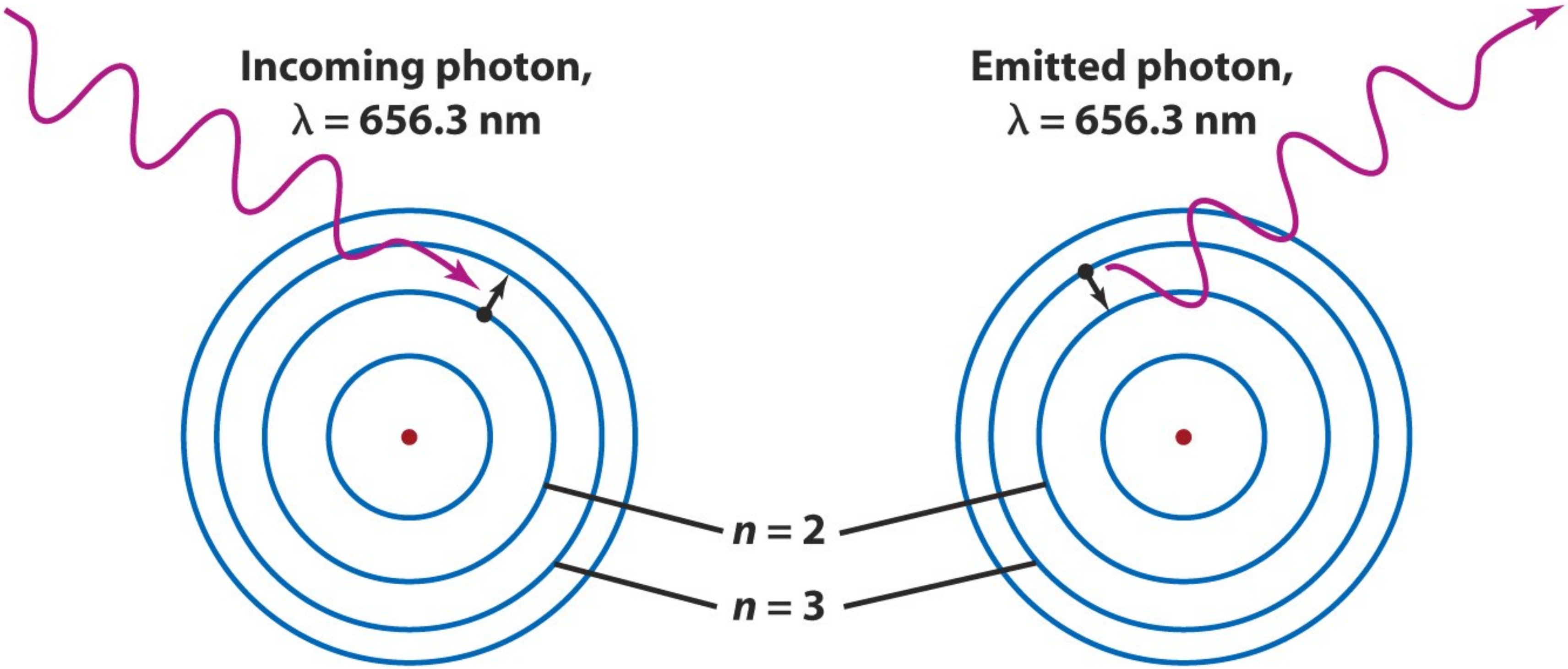


Kirchhoff's Laws



What is a photon a how light is generated ?

- as a kind of conceptual skeleton.



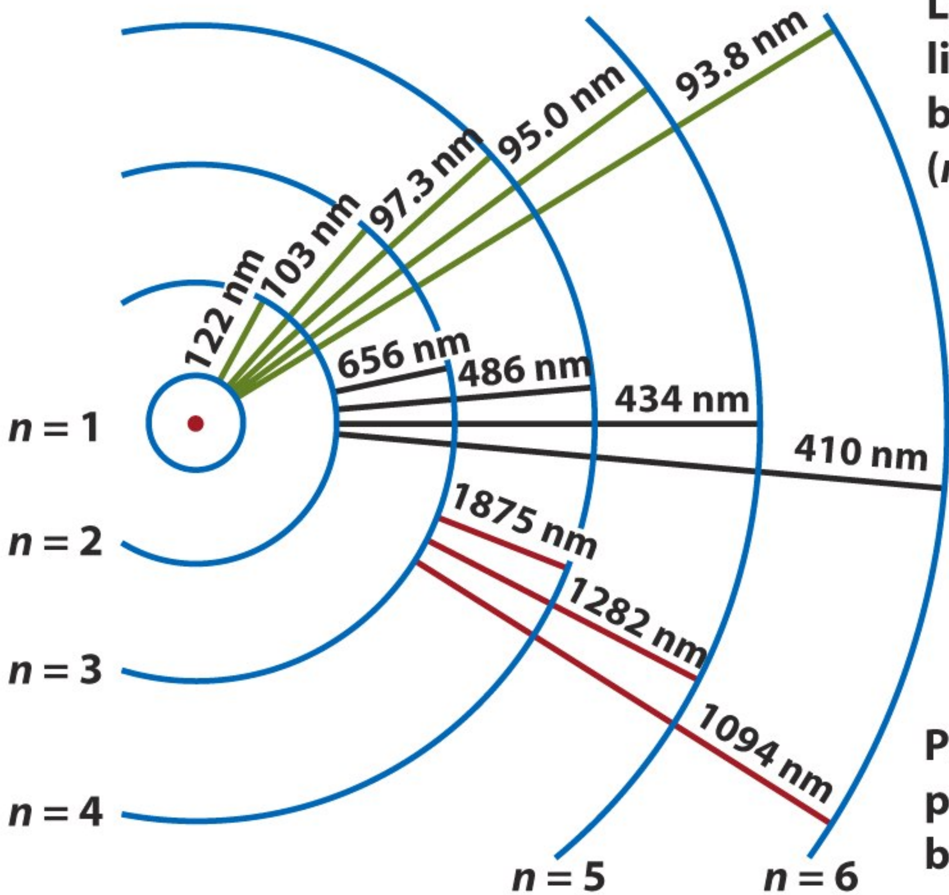
(a) Atom absorbs a 656.3-nm photon; absorbed energy causes electron to jump from the $n = 2$ orbit up to the $n = 3$ orbit

(b) Electron falls from the $n = 3$ orbit to the $n = 2$ orbit; energy lost by atom goes into emitting a 656.3-nm photon

What is known of [photons] comes from observing the results of their being created or annihilated

Eugene Hecht

What is a photon a how light is generated ?



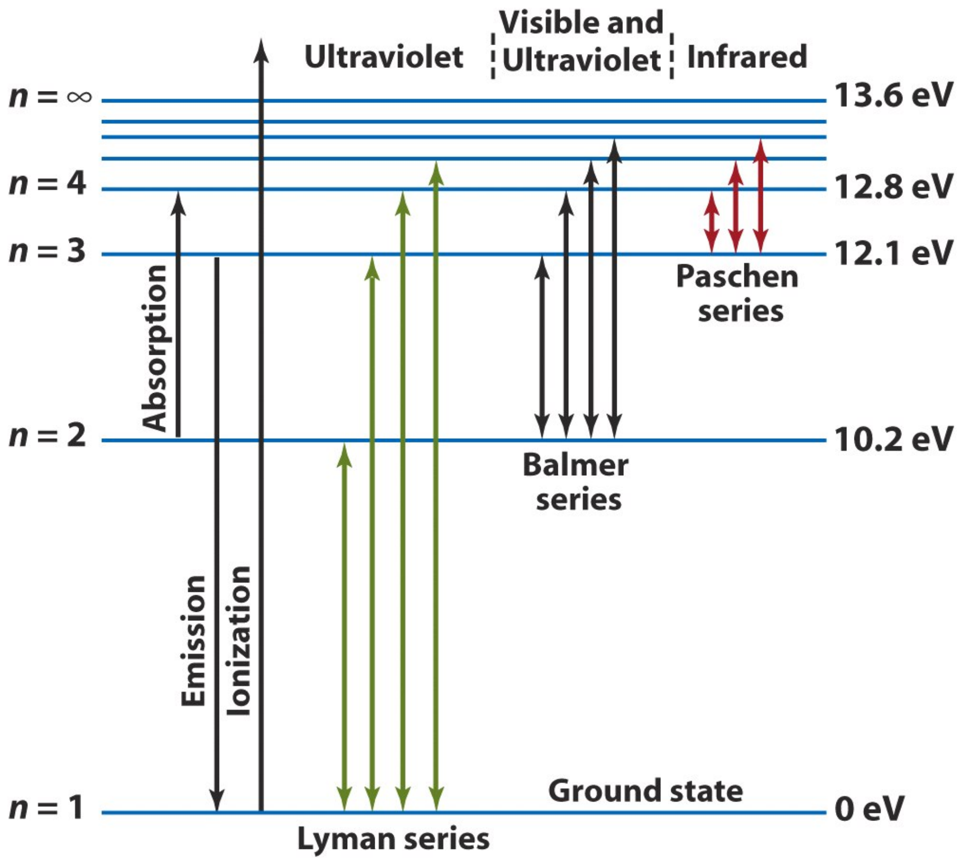
Lyman series (ultraviolet) of spectral lines: produced by electron transitions between the *n* = 1 orbit and higher orbits (*n* = 2, 3, 4, …)

Balmer series (visible and ultraviolet) of spectral lines: produced by electron transitions between the *n* = 2 orbit and higher orbits (*n* = 3, 4, 5, …)

Paschen series (infared) of spectral lines: produced by electron transitions between the *n* = 3 orbit and higher orbits (*n* = 4, 5, 6, …)

What is a photon and how light is generated ?

Bohr's formula for hydrogen wavelengths



$$\frac{1}{\lambda} = R\left(\frac{1}{N^2} - \frac{1}{n^2}\right)$$

N = number of inner orbit

n = number of outer orbit

R = Rydberg constant (1.097×10^7 m^{-1})

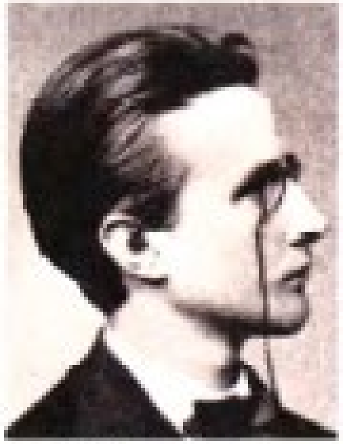
λ = wavelength of emitted or absorbed photon

What is a photon a how light is generated ?

- What makes this process special is the **quantized nature of the electron orbits.** As we said, there are only **a special set of allowed orbits for an electron**. Thus, there are only a special allowed sets of energy for an electron. Each orbit has a particular energy associated with it--we say the energy of an orbit is "quantized".

- To move from one orbit outwards to another, the electron ***must absorb exactly the right amount of energy--it cannot absorb more or less, it has to be exactly right***

- The electron must absorb a *photon* of light with **exactly the right amount of energy.**

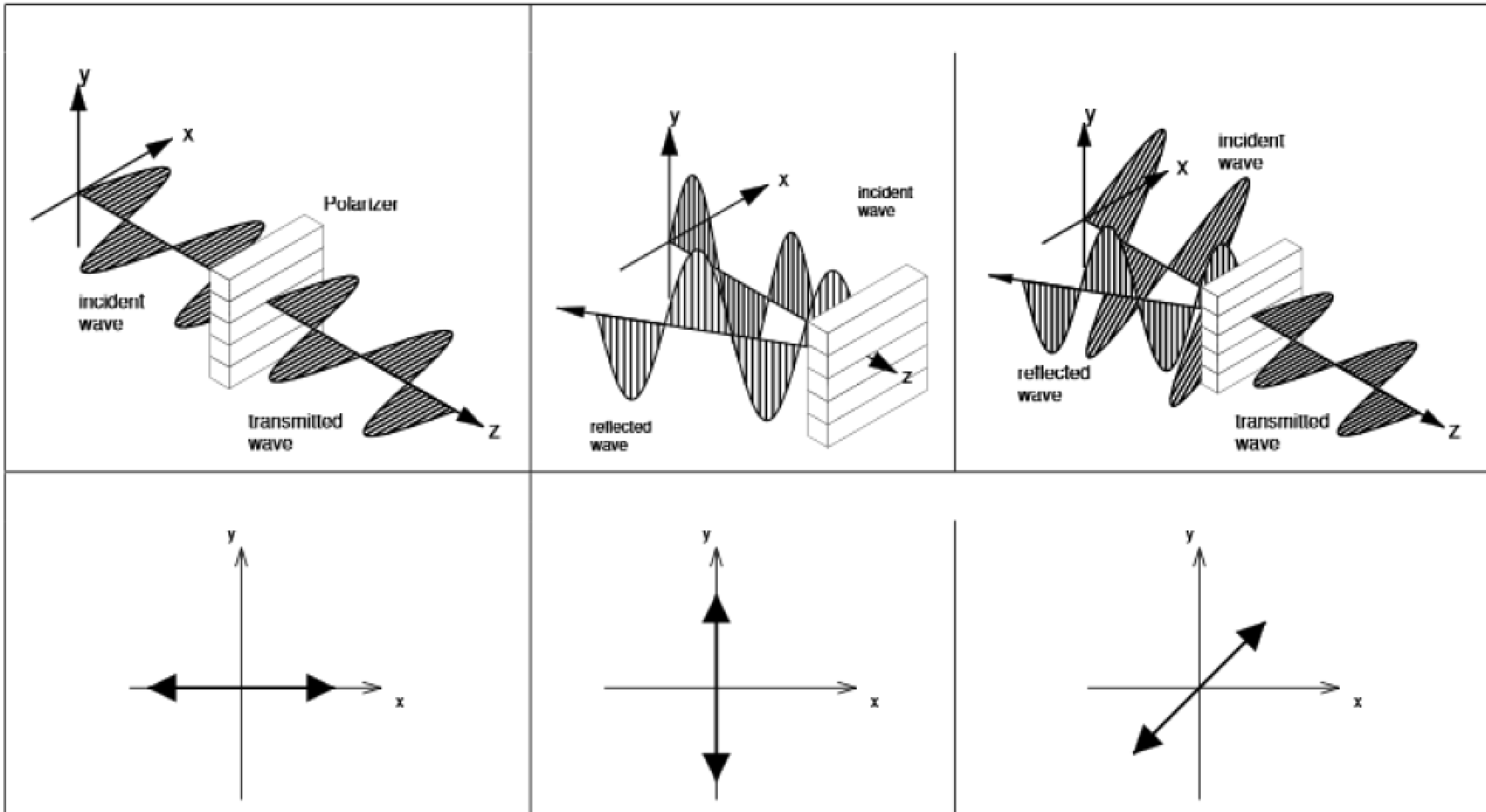


Max Planck (1858-1947) suggested that the emission and absorption of light by matter takes the form of quanta of energy.

Albert Einstein (1879-1955) advanced the hypothesis that light itself comprises quanta of energy.

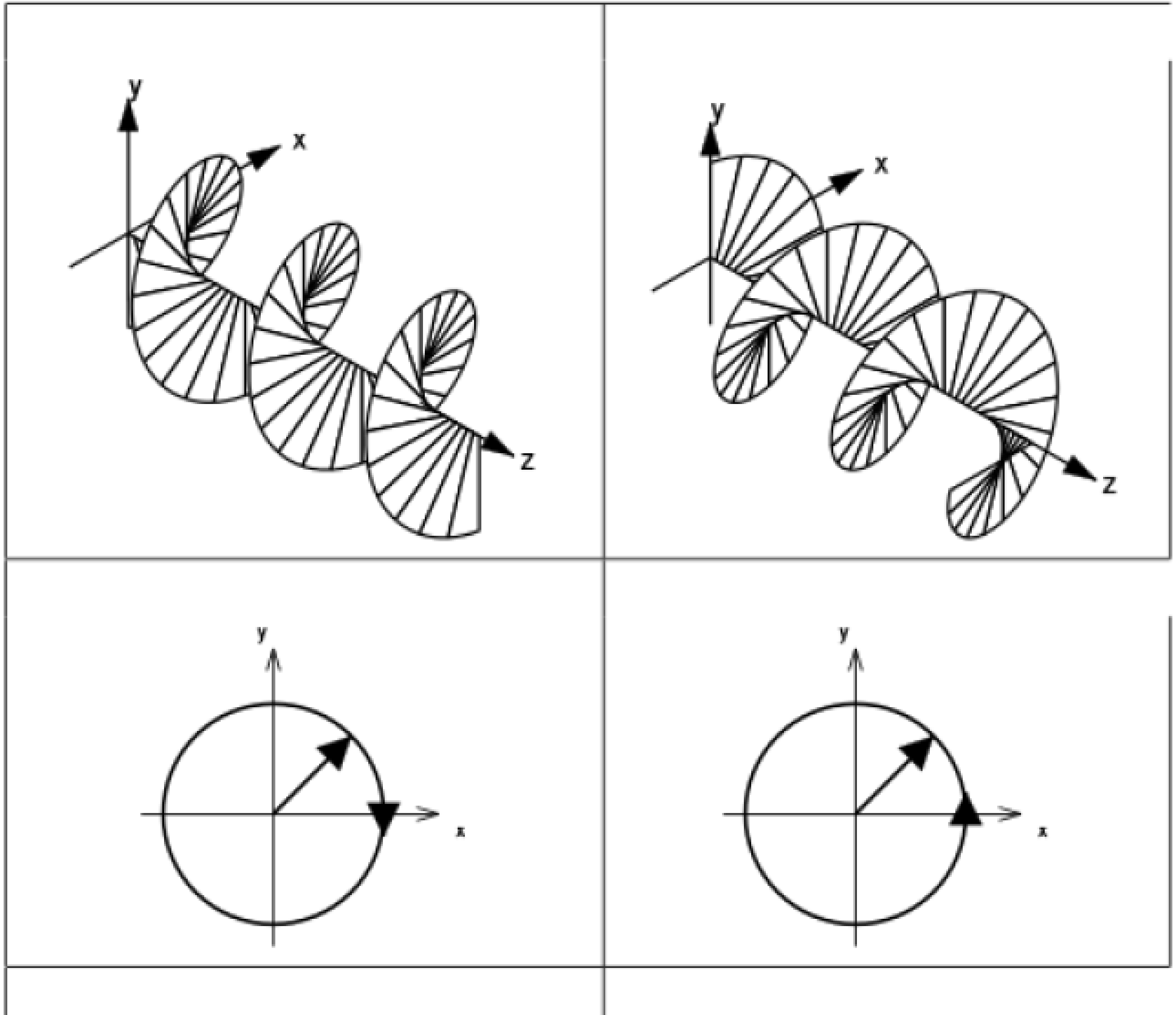
Linearly polarized light incident to a polarizer

- light is travelling in the positive *z*-direction, with angular frequency ω and wavevector $\mathbf{k} = (0,0,k)$, where the wavenumber $k = \omega/c$.



Horizontal polarization | Vertical Polarization | Arbitrary linear polarization

Circular polarization



Right circular polarization | Left circular polarization

Photon polarization

- As indicated earlier, light is characterized by a set of modes of **different frequencies, directions, and polarizations,** each occupied by a number of photons. For each monochromatic plane wave traveling in some direction, there **are two polarization modes**

$$\mathbf{E}(\mathbf{r}, t) = \sum_{\mathbf{q}} A_{\mathbf{q}} U_{\mathbf{q}}(\mathbf{r}) \exp(j2\pi\nu_{\mathbf{q}} t)\, \hat{\mathbf{e}}_{\mathbf{q}}.$$

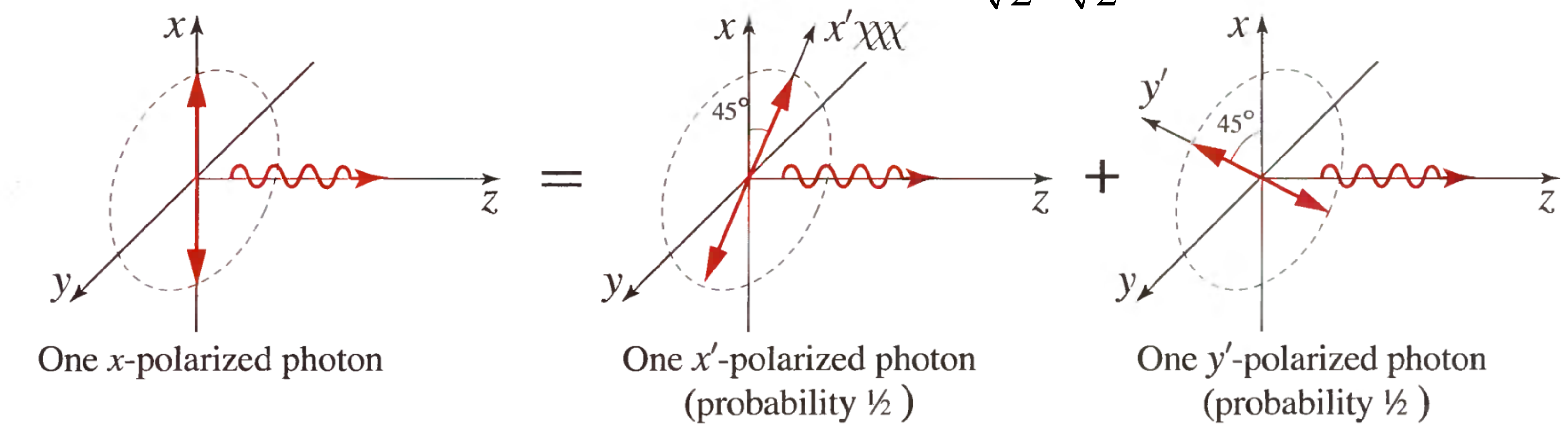
- Since the polarization modes of free space are **degenerate,** they are not unique. One may use modes with linear polarization in the x and y directions, linear polarization in two other orthogonal directions, say x' and y', or right- and left-circular polarizations.

$$A_{x'} = \frac{1}{\sqrt{2}}(A_x - A_y), \qquad A_{y'} = \frac{1}{\sqrt{2}}(A_x + A_y),$$

The components Ax, Ay are transformed from one coordinate system to another like ordinary Jones vectors, and the new components represent complex probability amplitudes in the new modes. ***Thus, a single photon may exist, probabilistically, in more than one mode.***

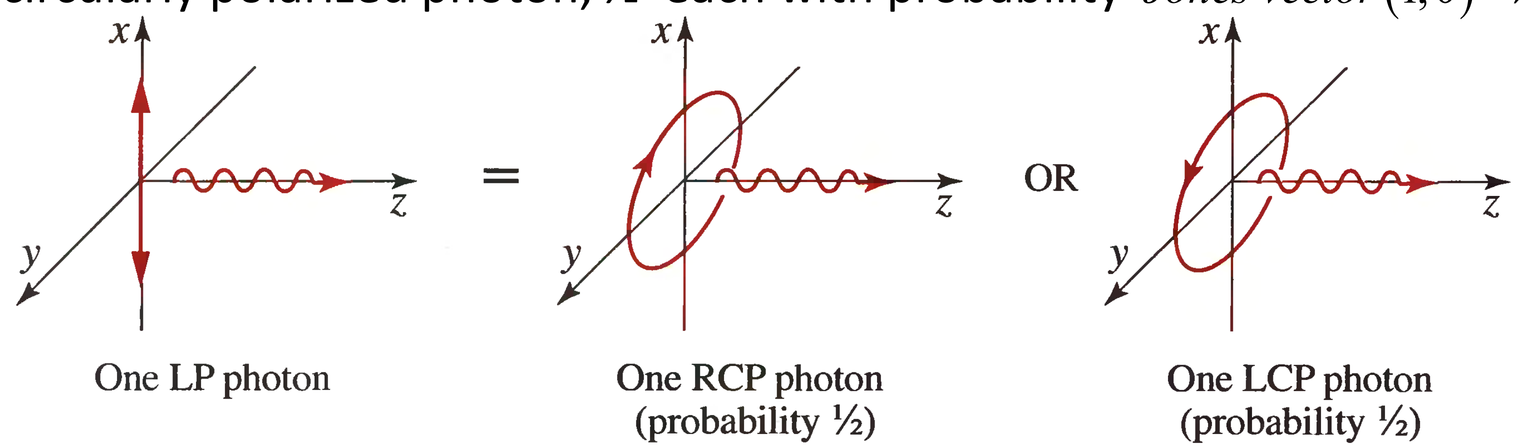
Photon polarization

- A photon in the x linear polarization mode is the same as a photon in a superposition of the x' linear polarization mode and the y' linear polarization mode with probability ½ each. $\textit{Jones vector}\,(1,0) \rightarrow (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$



A linearly polarized photon is equivalent to the superposition of a right- and a left-circularly polarized photon, ½ each with probability $\textit{Jones vector}\,(1,0) \rightarrow \frac{1}{\sqrt{2}}(1 \pm j)$

.



6 common examples of normalized Jones vectors.

Polarization	Corresponding Jones vector
Linear polarized in the x-direction Typically called 'Horizontal'	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
Linear polarized in the y-direction Typically called 'Vertical'	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
Linear polarized at 45° from the x-axis Typically called 'Diagonal' L+45	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
Linear polarized at −45° from the x-axis Typically called 'Anti-Diagonal' L-45	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
Right Hand Circular Polarized Typically called RCP or RHCP	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ +i \end{pmatrix}$
Left Hand Circular Polarized Typically called LCP or LHCP	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$

Jones matrices for common optical elements

Optical Element	Jones matrix
horizontal linear polarizer	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
vertical linear polarizer	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
linear polarizer at θ	$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$
quarter wave plate (fast axis vertical)	$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$
quarter wave plate (fast axis horizontal)	$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$

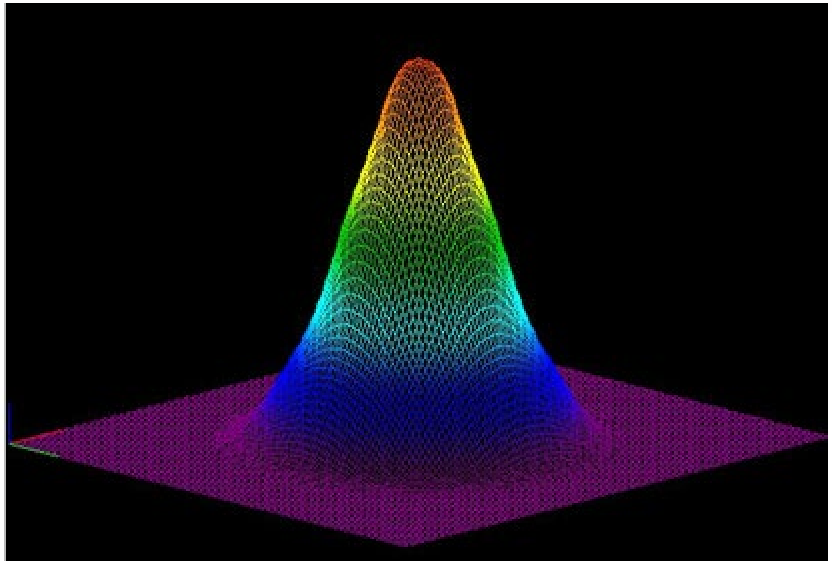
When light crosses an optical element the resulting polarization of the emerging light is found **by taking the product of the Jones matrix of the optical element and the Jones vector of the incident light**. Light which is randomly polarized, partially polarized, or incoherent must be treated using Mueller calculus.

Photon position

- When a photon impinges on a detector of small area **dA** located normal to the direction of propagation at the position r, its indivisibility causes it to be either **wholly detected or not detected at all**

The probability $p(\mathbf{r})dA$ of observing a photon at a point $\mathbf{r}$ within an incremental area dA, at any time, is proportional to the local optical intensity $I(\mathbf{r}) \propto |U(\mathbf{r})|^2$, so that

$$p(\mathbf{r})\, dA \propto I(\mathbf{r})\, dA.$$



$$U(\mathbf{r})\ \exp(j2\pi\nu t)$$

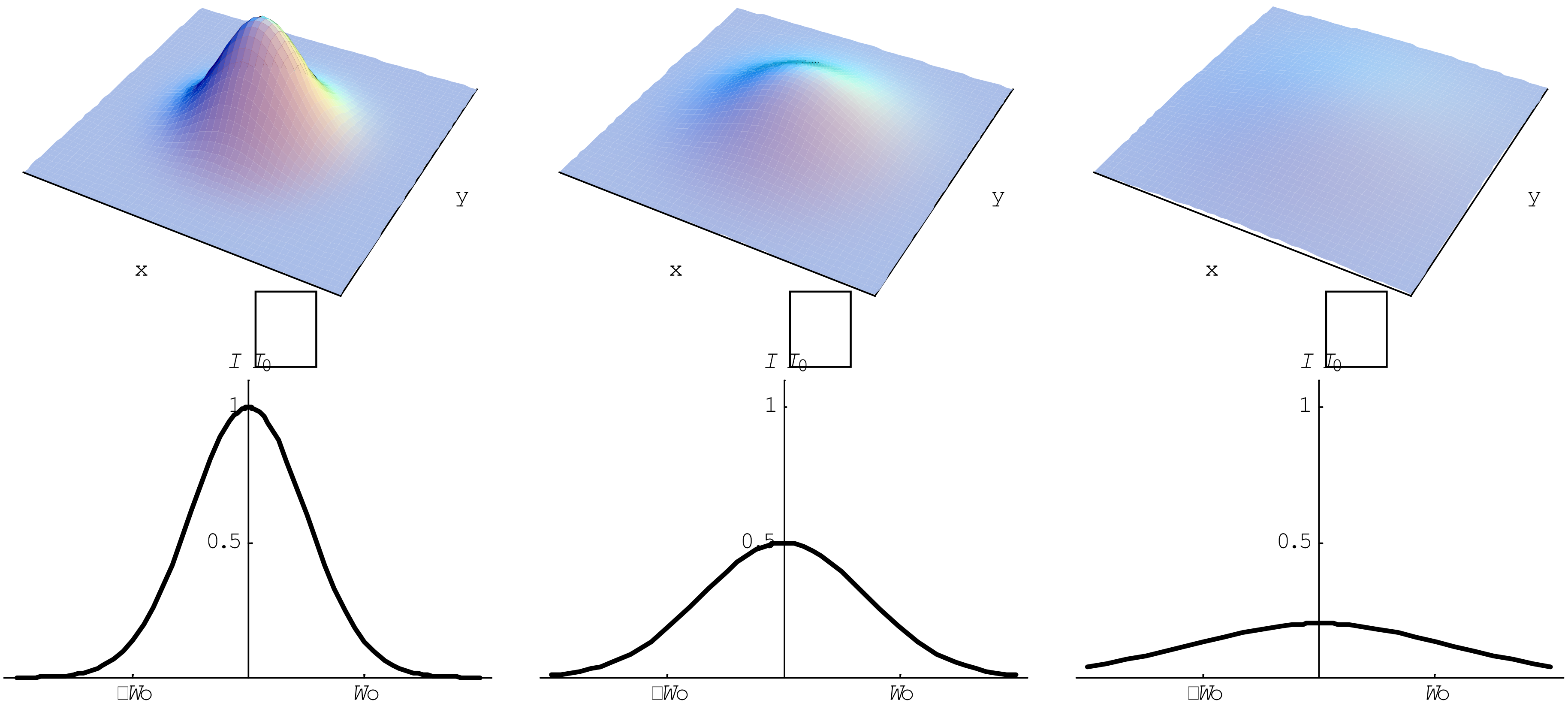
The photon is therefore **more likely to be found at those locations where the intensity is high.**

Optical photons behave as extended and localized entities. This behavior is called **wave particle duality**.

The localized nature of photons becomes evident when they are detected.

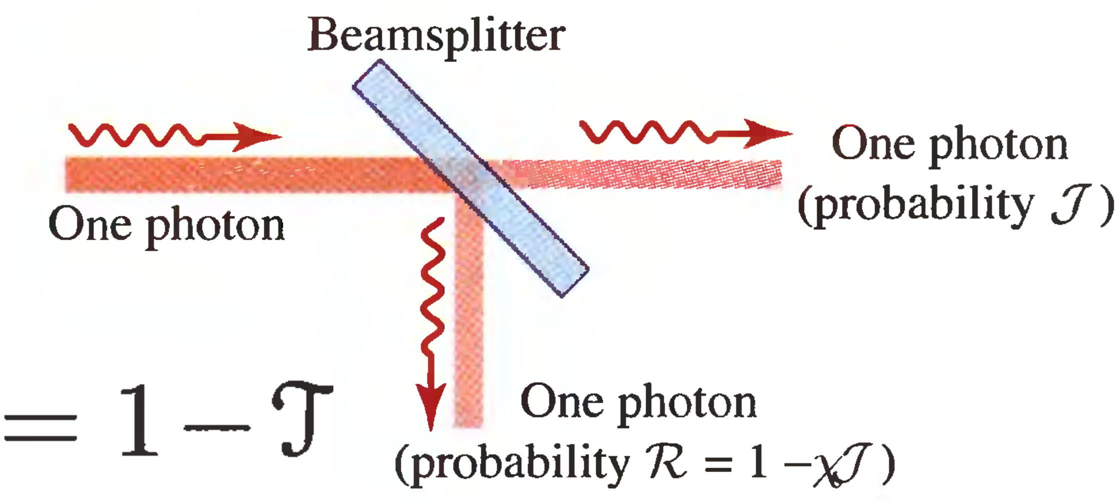
Gaussian Beam Intensity Plots

$$\frac{I(\rho,z)}{I_0} = \left[\frac{W_0}{W(z)}\right]^2 \exp\left[-\frac{2\rho^2}{W^2(z)}\right]$$



Transmission of a Single Photon Through a Beamsplitter

- A photon is indivisible, it must choose between the two possible directions permitted by the beamsplitter.



intensity reflectance $\mathcal{R} = 1 - \mathcal{T}$

intensity transmittance $\mathcal{T}$

The probability that the photon is transmitted is proportional intensity transmittance

$$\mathcal{T} = I_t / I$$

The probability that it is reflected $$1 - \mathcal{T} = I_r / I.$$

Photon Momentum

In photon optics, the linear momentum of a photon is $\mathbf{p} = (E/c)\hat{\mathbf{k}}$ where $E = \hbar\omega = \hbar c k$ is the photon energy.

- Photons carry momentum

$$\left|\vec{p}\right| = \frac{h}{\lambda}$$

- **Change in momentum corresponds** to the **force** and it can be calculated by the difference is **momentum flux S** between entering and leaving a object

$$\vec{F} = \frac{n}{c}\iint\left(\vec{S}_{in} - \vec{S}_{out}\right)dA$$

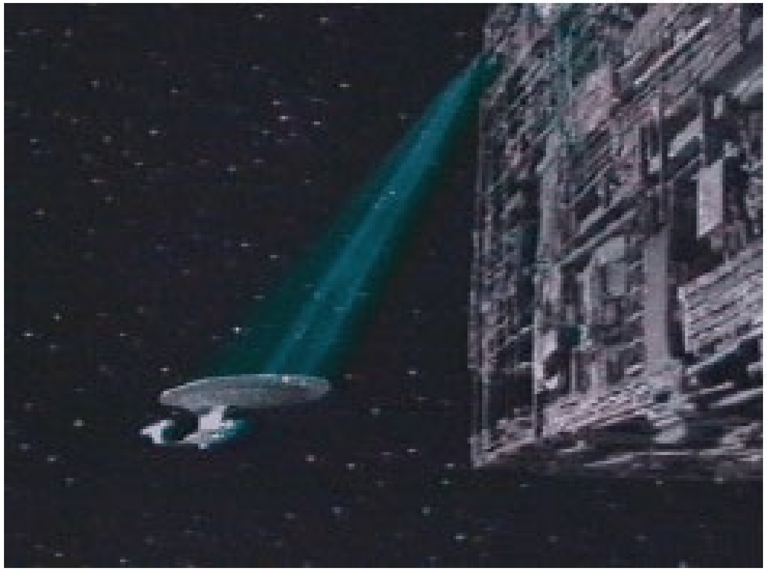
- Applying this formula to a 100% reflecting mirror reflecting a 60W lamp gives a pressure of:

$$\vec{F} = 2\frac{n}{c}\iint\left(\vec{S}_{in}\right)dA$$

$$\vec{F} = 2\frac{n}{c}W = 4\times10^{-7}\,N$$

Photon Momentum

- momentum associated with a photon can be transferred to objects of finite mass, giving rise to a force and causing mechanical motion. As an example, light beams can be used to deflect atomic beams traveling perpendicularly to the photons. The term radiation pressure is often used to describe this phenomenon (pressure force/area).
- .
- Sunlight on earth 0.5 nN/cm^2



Gravity pulls on a 1 kg mirror with 9.8 N so the force of the photons is negligible.

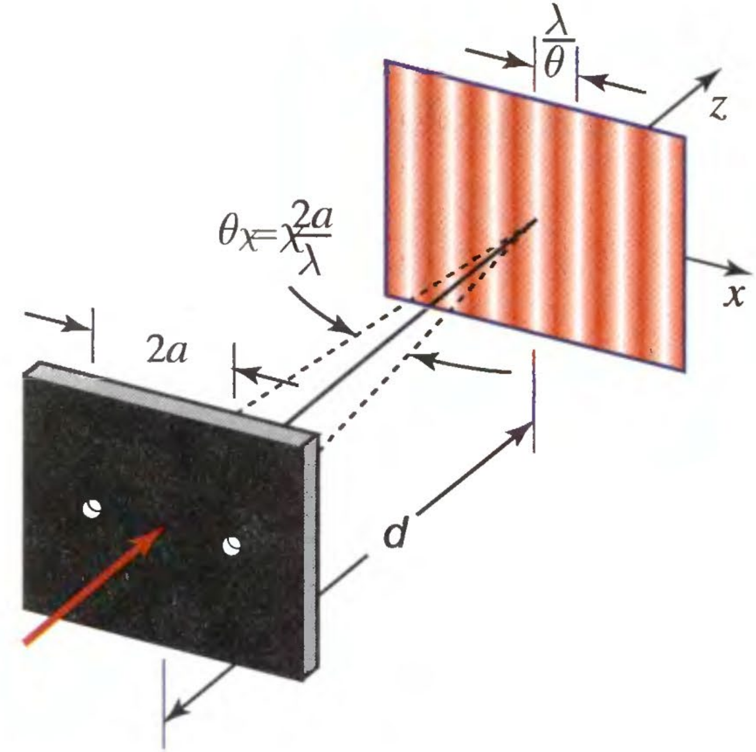
- However, if the same light is reflected by a object of 1 µg it can't be ignored!
- Using a laser on a microscopic particle will realize this situation

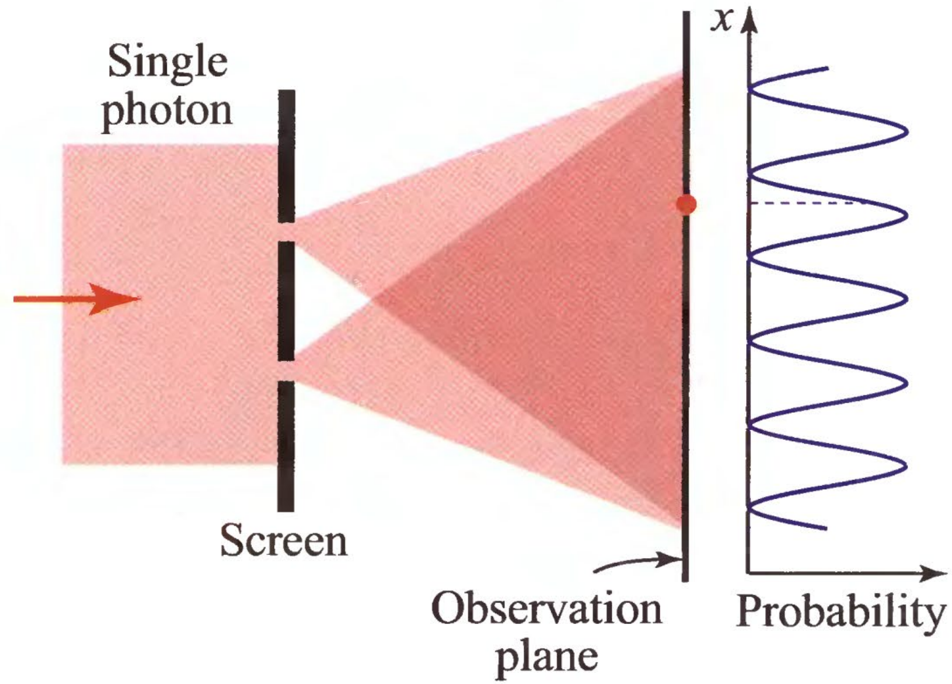
Physics of optical trapping

- The physics of the trapping mechanism is based on optical gradient and scattering forces arising from the interaction of strongly focused laser light with matter
- Simple models that explain optical trapping behavior can be applied in the **Mie scattering ($d >> \lambda$)** and the **Rayleigh scattering ($d << \lambda$)** regimes depending on the size of the particle relative to the wavelength of laser light
- A real optical tweezers typically works in the intermediate ($d \approx \lambda$) regime, requiring a rigorous application of complicated approaches such as Generalized Lorentz-Mie Scattering or T-Matrix theory (beyond the scope of this lecture!)

Photon Interference

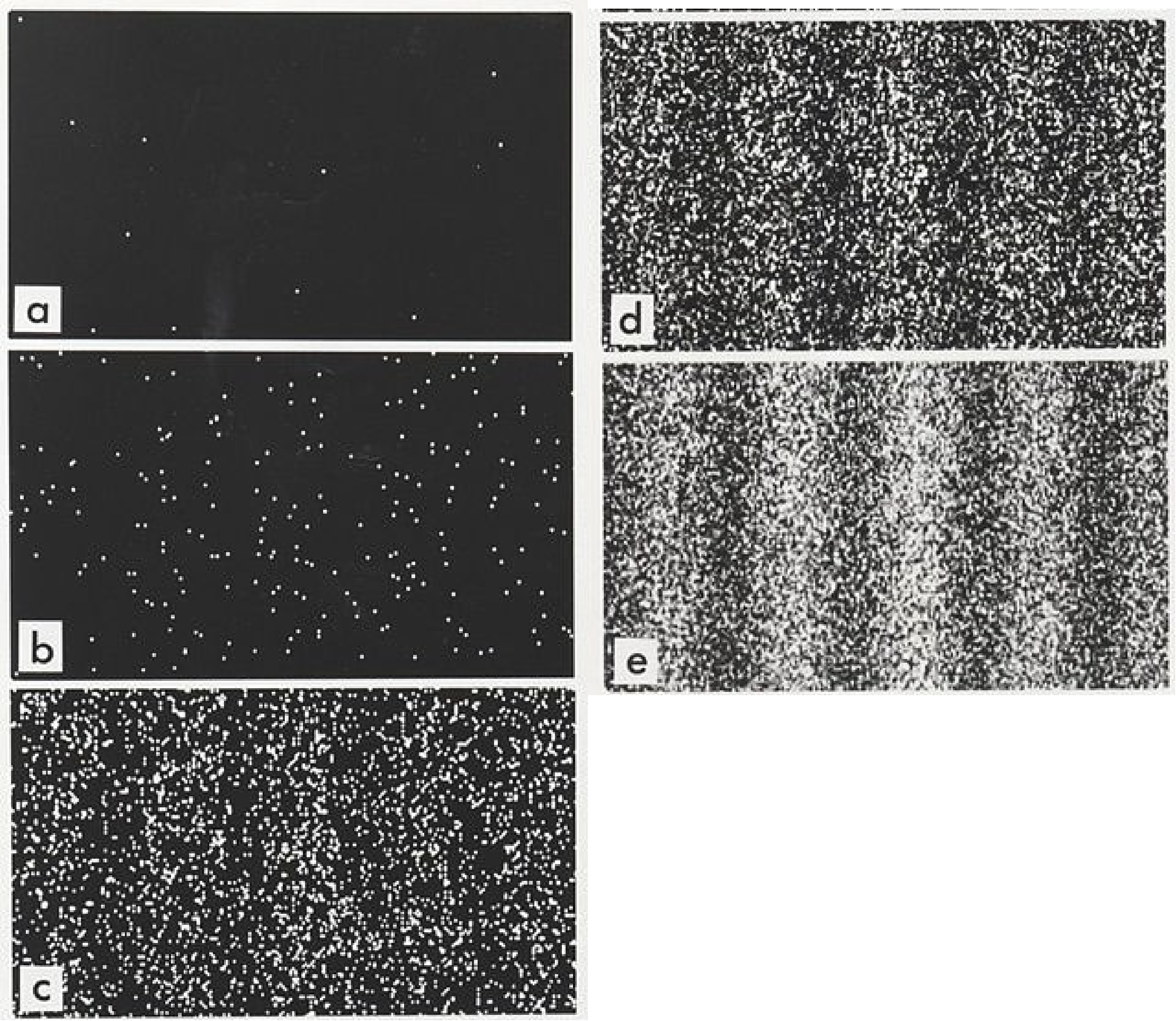
- Young's double-pinhole interference experiment is generally invoked to demonstrate the wave nature of light





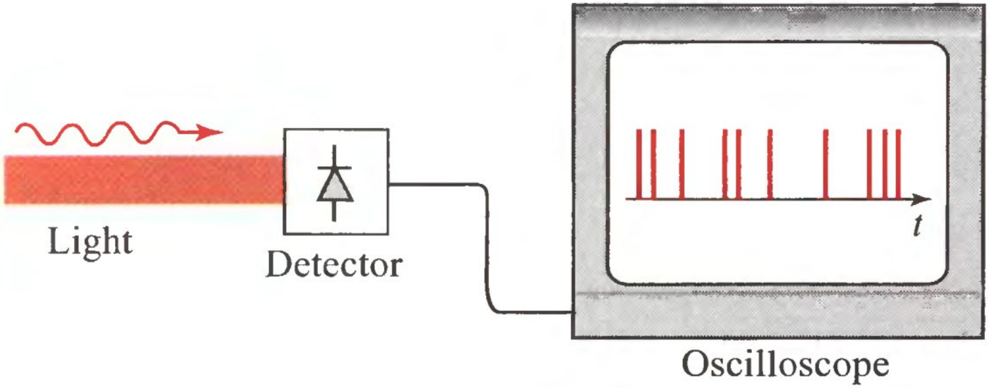
The occurrence of the interference results from the extended nature of the photon, which permits it to pass through both holes of the apparatus. This gives it knowledge of the entire geometry of the experiment when it reaches the observation plane, where it is detected as a single entity.

Photon Interference



Photon Streams

- The temporal pattern of such photon registrations can be highlighted by examining
- **the temporal and spatial behavior separately**. Consider the use of a detector with **good temporal resolution that integrates light over a finite area A**



$$P(t) = \int_A I(\mathbf{r}, t)\, dA.$$

Photon registrations at random localized instants of time for a detector that integrates light over an area A.

Mean Photon-Flux Density

Monochromatic light of frequency v and classical intensity I(r) (Watts/cm ^{2}) carries a mean photon-flux density

$$\phi(\mathbf{r}) = \frac{I(\mathbf{r})}{h\nu}.$$

Mean Photon Flux Density/ Mean Photon Flux

- Typical values of q; r for some common sources of light

Source	Mean Photon-Flux Density (photons/s-cm^2)
Starlight	10^6
Moonlight	10^8
Twilight	10^{10}
Indoor light	10^{12}
Sunlight	10^{14}
Laser light[a]	10^{22}

[a] A 10-mW He–Ne laser beam at $\lambda_o = 633$ nm focused to a 20-μm-diameter spot.

The mean photon flux Φ (units of photons /s) is obtained by integrating the mean photon-flux density over a specified area

$$\Phi = \int_A \phi(\mathbf{r})\, dA = \frac{P}{h\overline{\nu}},$$

As an example, 1 nW of optical power, at a wavelength λ 200 nm, delivers to an object an average photon flux $\Phi = 10^{9}$ photons per second.

Mean Number of photons

- The mean number of photons **n** detected in the area A and in the time interval T is obtained by multiplying the mean photon flux Φ in by the time duration

$$\overline{n} = \Phi T = \frac{E}{h\overline{\nu}},$$

Classical		Quantum	
Optical intensity	$I(\mathbf{r})$	Photon-flux density	$\phi(\mathbf{r}) = I(\mathbf{r})/h\overline{\nu}$
Optical power	P	Photon flux	$\Phi = P/h\overline{\nu}$
Optical energy	E	Photon number	$\overline{n} = E/h\overline{\nu}$

Time Varying Light

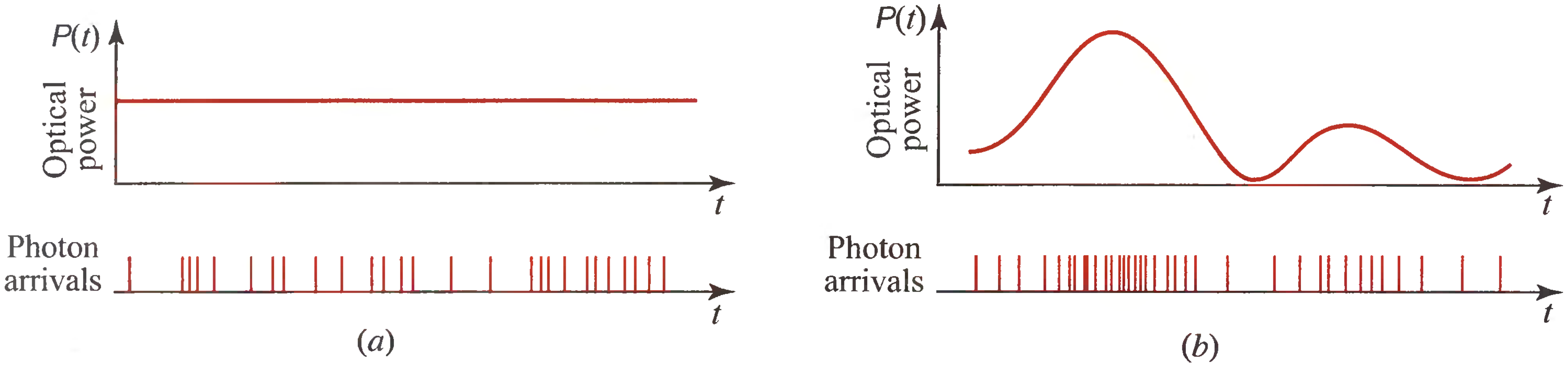
$$\phi(\mathbf{r}, t) = \frac{I(\mathbf{r}, t)}{h\overline{\nu}}.$$

$$\Phi(t) = \int_A \phi(\mathbf{r}, t)\, dA = \frac{P(t)}{h\overline{\nu}}, \quad \overline{n} = \int_0^T \Phi(t)\, dt = \frac{E}{h\overline{\nu}},$$

Randomness of Photon Flow

- For photon streams, the classical intensity **I(r,t)** determines the mean photon- flux density **ϕ(r,t).** The properties of the light source determine the fluctuations

- the times at which the photons are detected are random, their statistical behavior determined by the source,



(a) Constant optical power and the corresponding **random photon arrival times**.

(b) Time-varying optical power and the corresponding random photon arrival times

Randomness of Photon Flow

- An understanding of photon-number statistics is important for applications such as reducing noise in weak images and optimizing optical information transmission.

- **Coherent light** has a constant optical power P. The corresponding mean photon flux $\Phi = P/h\nu$ (photons/s) is also constant, but the actual times of registration of the photons are random**. An expression for the probability distribution p(n) can be derived under the assumption that the registrations of photons are statistically independent. The result is the Poisson distribution**

$$p(n) = \frac{\bar{n}^n \exp(-\bar{n})}{n!}, \qquad n = 0, 1, 2, \ldots$$

- It is not difficult to show in and that the mean of the Poisson distribution is indeed n and its variance is equal to its mean:

$$\sigma_n^2 = \bar{n}.$$

Signal to Noise Ratio

- **The randomness of the number of photons** constitutes a fundamental source of noise that we have to account for when using light to transmit a signal. Representing the mean of the signal as **n,** and its noise by **the root mean square value is** σn, a useful measure of the performance of light as an information-carrying medium is the signal-to-noise ratio (SNR). The SNR of the random number n is defined as

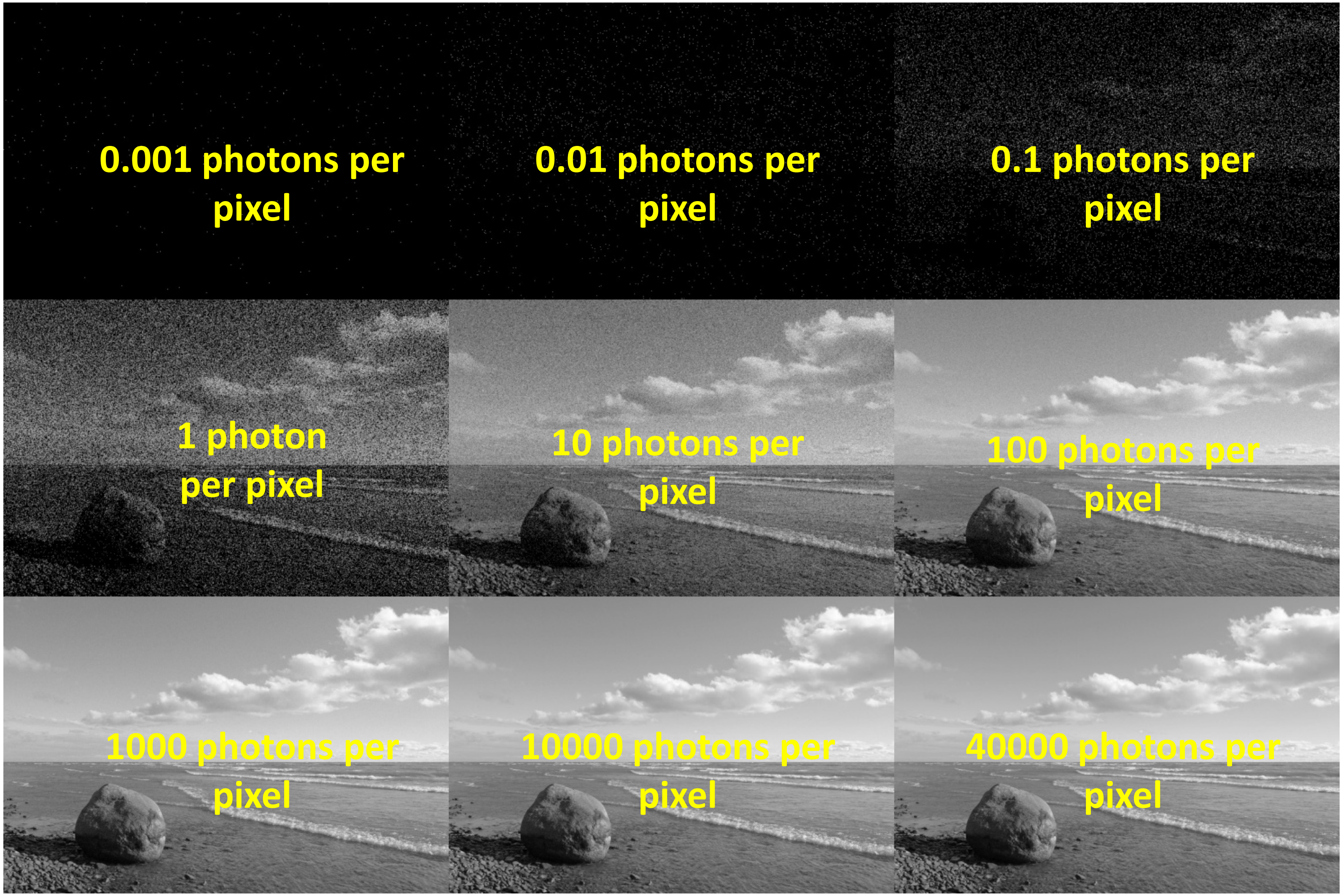
$$\mathrm{SNR} = \frac{(\text{mean})^2}{\text{variance}} = \frac{\bar{n}^2}{\sigma_n^2}.$$

For the Poisson distribution

$$\mathrm{SNR} = \bar{n},$$

so that the signal-to-noise ratio **increases linearly with the mean number of photon counts.**

Photon Shot Noise/ Exposure dependent



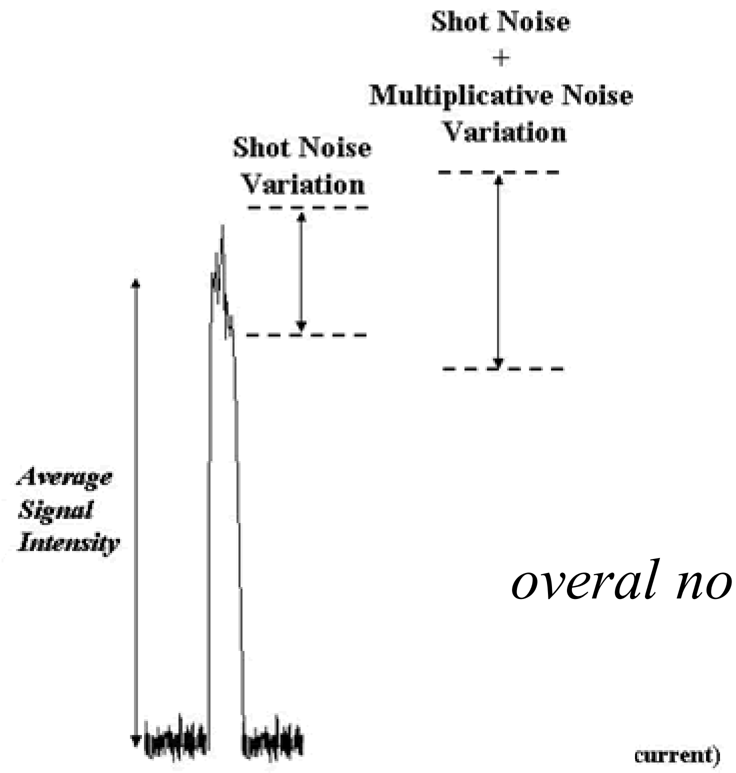
A photon noise simulation, using a sample image as a source and a per-pixel Poisson process to model an otherwise perfect camera (quantum efficiency = 1, no read-noise, no thermal noise, etc).

Noise Sources of a Detector

Photon Shot Noise – Counting statistics of the signal photons

-Originates from the Poisson distribution of signal photons as a function of time

-Random arrival of photons and electron is governed by Poisson distribution



- Dark Current Noise – Counting statistics of spontaneous electron generated in the device
- Johnson Noise – Thermally induced current in the transimpedence amplifier

$$overal\ noise = \sqrt{(readnoise)^2 + (darknoise)^2 + (shotnoise)^2}$$

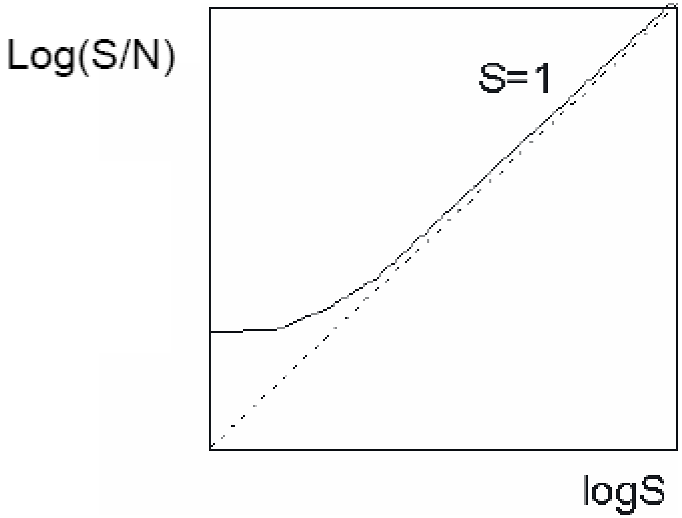
Photon Shot Noise

- Shot noise is white noise, just like Johnson noise. Does not exist unless current is driven through the device.
- This is termed **"white noise"** why?
- Because, like in white light**, all *frequencies* are equally represented**

- Standard deviation (or noise) and the photon noise limited Signal-to-Noise-Ratio (SNR) associated with detecting a mean of 'N' photons are given by

$$Noise\,(photon) \approx \sqrt{N}$$

$$SNR(photon) = \frac{Signal}{Noise} \approx \frac{N}{\sqrt{N}} = \sqrt{N}$$



- Means to **enhance** S/N
  - Signal averaging: internally or externally
  - Signal smoothing: boxcar averaging, moving average, polynomial smoothing (**keyword: convolution**)
  - Filtering in the frequency domain: